1. Consider the following spring-mass-damper system. We measure the displacement $y$ of the mass from a static equilibrium position (so that the gravitational force is balanced by the equilibrium spring deflection). An external force $f$ is applied and treated as input. Suppose that $m=1 \text{ kg}$, $b=2 \text{ Nm/s}$, and $k=2 \text{ N/m}$.

![Spring-mass-damper system diagram]

a. What is the governing ordinary differential equation?

b. Obtain state-space representation of the system dynamics with input $f$ and output $y$.

c. Build a SIMULINK model to simulate the system. Print your model. Print the response of $y(t)$ corresponding to an input $f(t) = 1 + \sin(\pi t)$.


Given inputs $f_1 = \sin(\pi t)$ and $f_2 = \frac{1}{t} + 2$, simulate and obtain the responses $y_1$, $y_2$.

Let $f_3 = \alpha f_1 + \beta f_2$. Simulate and obtain $y_3$ corresponding to $f_3$. Check if $y_3 = \alpha y_1 + \beta y_2$.

Plot $y_3$ and $\alpha y_1 + \beta y_2$ on the same plot for different combinations:

1) $\alpha = 3$, $\beta = 0$; 2) $\alpha = 0$, $\beta = 5$; 3) $\alpha = 3$, $\beta = 6$; 4) $\alpha = 2.5$, $\beta = -6$.

Also plot $y_3 - (\alpha y_1 + \beta y_2)$ for each scenario. [Make sure you do not need to modify your simulink model for different values of $\alpha$ and $\beta$.]
2. Assume the following differential equation:

\[ 4\ddot{x} + 5x\dot{x} - 10x = u(t) \]

a. Obtain state-space representation of the system dynamics with input \( u \) and output \( x \).

b. Build a SIMULINK model to simulate the system. Print your model. Print the response of \( y(t) \) corresponding to input \( u(t) = 6 \cos\left(\frac{\pi}{6} t + 5\right) \sin(2t) + e^{-3t} \).

\[ \text{c. Is the dynamical system linear? Why? Verify your answer using simulation.} \]

Given inputs \( f_1 = \sin(2t) \) and \( f_2 = \frac{1}{t} + 2 \). Simulate and obtain the responses \( y_1, y_2 \).

Let \( f_3 = \alpha f_1 + \beta f_2 \). Simulate and obtain response \( y_3 \) corresponding to \( f_3 \). Check if \( y_3 = \alpha y_1 + \beta y_2 \).

Plot \( y_3 \) and \( \alpha y_1 + \beta y_2 \) on the same plot for different combinations:

1) \( \alpha = 3, \ \beta = 0 \); 2) \( \alpha = 0, \ \beta = 5 \); 3) \( \alpha = 3, \ \beta = 6 \); 4) \( \alpha = 2.5, \ \beta = -6 \);

Also plot \( y_3 - (\alpha y_1 + \beta y_2) \) for each scenario.

[Make sure you do not need to modify your SIMULINK model for different values of \( \alpha \) and \( \beta \)]