Novel $\mathcal{L}_1$ Adaptive Control Approach to Autonomous Aerial Refueling with Guaranteed Transient Performance

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Abstract—Autonomous aerial refueling autopilot design is addressed in this paper using a novel $\mathcal{L}_1$ neural network based adaptive control approach. The main advantage of the new approach is its ability to guarantee transient performance with desired specifications for system's both input and output signals by systematic choice of design parameters. Simulation results for a fighter aircraft model illustrate the benefits of this control approach.

I. INTRODUCTION

With the advent of unmanned flight vehicles, safe and reliable Autonomous Aerial Refueling (AAR) capabilities become a necessity. As a piloted task, aerial refueling has proven to be extremely difficult due to the aerodynamic influence of the receiver aircraft on the drogue. Several methods have been practiced for obtaining accurate measurements of the drogue, like use of global positioning system (GPS) in [7], [8], visual servoing with pattern recognition in [10], [11], [12] and vision-based navigation systems in [9]. In [13], GPS based sensor has been used with machine vision-based sensors to achieve accurate positioning and capturing. The situation is also complicated by the fact that the receiver aircraft has uncertainties in its dynamics due to the aerodynamic influences from the tanker. Towards this end, disturbance rejection methods have been considered in [14], while in [15] methods have been suggested for improving the disturbance modelling. In [16], differential game theory is used to define an optimal reference model that the receiver aircraft tracks via an adaptive controller.

There are currently two aerial refueling methods in use: the flying boom approach and probe-and-drogue approach. In this paper, the probe-and-drogue aerial refueling system is considered for the nonlinear longitudinal model of the aircraft under the assumption that the drogue coordinates are measured with sufficient accuracy. It is assumed that the tanker is in steady level flight and the coordinates of the drogue, moving in the vertical plane, are measured by one or other available method with sufficient accuracy. The autopilot design aims at achieving automatic maneuvers of the receiver aircraft (manned or unmanned) to send the probe to the close proximity of the moving drogue and maintain the probe there, without any specific knowledge

of the drogue dynamics. The coordinates of the drogue can be considered as reference inputs that the receiver aircraft needs to track. Model Reference Adaptive Control (MRAC) architecture has been intensively explored for solving this problem, however in such uncertain dynamic environment the transient performance of MRAC can be unpredictable.

Novel $\mathcal{L}_1$ neural network adaptive control method is considered in this paper for solving the AAR problem, [2]–[4]. The benefit of this new adaptive architecture is that it has guaranteed transient response in addition to stable tracking for system's both signals input and output simultaneously. This architecture includes a low-pass filter in the feedback loop that guarantees a low-frequency control signal, while increasing the adaptive gain.

The paper is organized as follows. In Section II, we give the problem formulation. In section III, the novel $\mathcal{L}_1$ control architecture is presented. Section IV discusses the performance results of the $\mathcal{L}_1$ adaptive control architecture. Simulation results for F-16 aircraft model are presented in section V.

II. PROBLEM FORMULATION

The longitudinal dynamics of the receiver aircraft with small angle assumption can be written as follows:

$$\begin{aligned} \dot{l}(t) &= V(t) - V_0 \\ \dot{V}(t) &= -g(\theta(t) - \alpha(t)) + X_{\delta_T} \delta_T(t) \\ &\quad - \frac{\rho V^2(t)}{2m} S C_D(M, \alpha) \eta(V, \alpha, q) \\ \dot{\theta}(t) &= q(t) \\ \dot{q}(t) &= M_{\delta_e} \delta_e(t) + \frac{\rho V^2(t)}{2I_y} S \bar{c} C_M(M, \alpha) \xi(V, \alpha, q) \\ \dot{h}(t) &= V_0(\theta(t) - \alpha(t)) \\ \dot{\alpha}(t) &= q(t) + Z_\alpha(\alpha(t) - \alpha_0), \end{aligned} \quad (1)$$

where $l(t)$ is the relative horizontal distance between the probe of the receiver aircraft and the tanker, $V(t)$ is the true airspeed, $\theta(t)$ is the pitch angle, $q(t)$ is the pitch rate, $h(t)$ is the vertical relative distance between the probe and the tanker, $\alpha(t)$ is the angle of attack, $C_D(M, \alpha)$ and $C_M(M, \alpha)$ are the non-dimensional drag coefficient and pitch moment coefficient respectively, m is the mass of the receiver aircraft, ρ is the air density, S is the wing area, $\bar{c}$ is the mean aerodynamic chord, I_y is the moment of inertia with respect to y axis, X_{δ_T} is the throttle effectiveness factor, M_{δ_e} is the elevator effectiveness factor, and Z_α is the aerodynamic coefficient of vertical force. The control

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inputs are the thrust $\delta_T(t)$ and the elevator deflection $\delta_e(t)$. The effects of the airflow from the tanker aircraft on the receiver dynamics are accounted in the terms $\eta(V, \alpha, q)$ and $\xi(V, \alpha, q)$. The receiver dynamics are trimmed around $(V_0, \alpha_0, \theta_0)$, while the tanker is assumed to be trimmed for steady-state level flight with the same trim conditions. The coordinate system is the body-fixed system of the tanker aircraft, with x direction pointing forward and z direction pointing downward. Let

$$\begin{aligned} x_1(t) &= l(t), \\ x_2(t) &= V(t) - V_0 \\ x_3(t) &= h(t) \\ x_4(t) &= V_0(\theta(t) - \alpha(t)) \\ x_5(t) &= \alpha(t) - \alpha_0 \\ x_6(t) &= q(t). \end{aligned}$$

Further, let

$$\begin{aligned} \Delta_1(V, \alpha, q) &= \frac{\rho V(t)^2}{2m} SC_D(M, \alpha) \eta(V, \alpha, q), \\ \Delta_2(V, \alpha, q) &= -\frac{\rho V(t)^2}{2I_y} S \bar{C}_M(M, \alpha) \xi(V, \alpha, q), \\ u_1(t) &= X_{\delta_T} \delta_T(t), \\ u_2(t) &= M_{\delta_e} \delta_e(t). \end{aligned}$$

The receiver dynamics in state-space form can be written as:

$$\begin{cases} \dot{x}(t) = A_p x(t) + B_p(u(t) - \Delta(x(t))), & x(0) = x_0 \\ y(t) = C_p^\top x(t), \end{cases} \quad (2)$$

where $x(t) = [x_1(t), \dots, x_6(t)]^\top$, $u(t) = [u_1(t), u_2(t)]^\top$ and $\Delta(x) = [\Delta_1(x), \Delta_2(x)]^\top$, while

$$\begin{aligned} A_p &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{g}{V_0} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -V_0 Z_\alpha & 0 \\ 0 & 0 & 0 & 0 & Z_\alpha & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ B_p &= \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad C_p = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned}$$

We further assume that from experimental evaluation some conservative knowledge is available about the Lipschitz constant of the uncertainties, so that for a compact set $x \in \mathcal{D}_x \subset \mathbb{R}^6$ one has the following uniform bounds:

$$\begin{aligned} |\Delta_i(x') - \Delta_i(x'')| &\leq L \|x' - x''\|, \quad i = 1, 2, \\ \max_{i=1,2} |\Delta_i(0)| &\leq B, \quad i = 1, 2, \end{aligned}$$

where $L > 0$ and B are known.

The control objective is to use elevator and throttle feedback to fly the receiver aircraft in finite time into a prespecified neighborhood of the drogue, within which the aerial refueling can be executed. Let $r(t) = [x_d(t) \quad z_d(t)]^\top$ specify the coordinates of the drogue center with respect to the coordinate system associated with the tanker. The refueling can start at any time instant $t' \leq T_f$, if $\|y(t') - r(t')\|_2 \leq r_d$, where r_d is the radius of the drogue, and T_f is a prescribed time for the drogue to capture the probe.

The main challenge of the aerial refueling problem is associated with the aircraft position control within finite time in the presence of the tanker airflow. To ensure that the receiver aircraft can reach the drogue within the prescribed finite time T_f and execute safe and reliable refueling maneuver, the autopilot needs to have guaranteed transient response in highly uncertain dynamic environment. Let the transient performance specifications for such a maneuver be given by a strictly proper and stable $D(s)$. Thus, the autopilot needs to ensure

$$y(s) \approx D(s)r(s),$$

where $y(s), r(s)$ are Laplace transformation of $y(t), r(t)$ respectively.

III. CONTROL DESIGN

In this Section, we present the novel adaptive control architecture from [2]–[4] that has guaranteed transient performance. Consequently, it provides a suitable architecture for solving the AAR problem.

We consider the following control structure:

$$u(t) = u_{lin}(t) + u_{ad}(t), \quad (3)$$

where $u_{lin}(t)$ is the nominal linear control signal that yields the desired performance in the absence of uncertainties, while $u_{ad}(t)$ is the adaptive augmentation. The linear controller $u_{lin}(t) = -K^\top x(t)$ is designed using conventional methods of LQR or pole placement such that $K^\top \in \mathbb{R}^{2 \times 6}$ satisfies $A_m = A_p - B_p K^\top$, where the Hurwitz matrix $A_m \in \mathbb{R}^{6 \times 6}$ is the system matrix of the state space representation of the desired transfer function $D(s)$. With the control signal in (3), the closed loop dynamics for the system in (2) can be rewritten as

$$\begin{aligned} \dot{x}(t) &= A_m x(t) + B_p(u_{ad}(t) - \Delta(x(t))), \quad x(0) = x_0 \\ y(t) &= C_p^\top x(t). \end{aligned} \quad (4)$$

Following the approach in [6], we consider linear parametrization of the system uncertainties in (2) over a compact set by a neural network:

$$\Delta(x) = W^\top \Phi(x) + \epsilon(x), \quad \|\epsilon(x)\| \leq \epsilon^*, \quad x \in \mathcal{D}_x, \quad (5)$$

where $\Phi(x)$ is a vector of suitably chosen Gaussian basis functions. Let $\Phi(x)$ be of dimension $p \times 1$, while W be defined as

$$W = \begin{bmatrix} W_1 & W_2 \end{bmatrix},$$

where W_1, W_2 are $p \times 1$ vectors. We further assume that a compact convex set Θ is known apriori such that $W_i \in \Theta$. Following the approach in [2]–[4], we consider the following companion model:

$$\begin{aligned}\dot{\hat{x}}(t) &= A_m \hat{x}(t) + B_p(u_{ad}(t) - \hat{W}^\top(t)\Phi(x(t))) \\ \hat{y}(t) &= C_p^\top \hat{x}(t),\end{aligned}\quad (6)$$

with $\hat{x}(0) = x_0$, where $\hat{W}(t)$ are the parameter estimates defined via the Projection operator [5]:

$$\dot{\hat{W}}(t) = -\Gamma \text{Proj}(\dot{W}(t), \Phi(x)\hat{x}^\top(t)PB_p), \quad (7)$$

in which $\tilde{x}(t) = x(t) - \hat{x}(t)$, $\Gamma = \Gamma_c \mathbb{I} \in \mathbb{R}^{p \times p}$, $\Gamma_c > 0$ is the adaptive gain, while $P = P^\top > 0$ is the solution of the algebraic Lyapunov equation $A_m^\top P + PA_m = -Q$ for some positive definite $Q > 0$.

Using standard Lyapunov arguments, one can prove that the tracking error $\tilde{x}(t) = x(t) - \hat{x}(t)$ and parameter error $\tilde{W}(t) = \hat{W}(t) - W$ are ultimately bounded, irrespective of the control signal $u_{ad}(t)$. However, to ensure stability of the entire system, one needs to specify the control signal $u_{ad}(t)$ and prove that one of the two systems, either (4) or (6), remains bounded with it.

Following the approach in [2]–[4], we consider the filtered adaptive control signal:

$$u_{ad}(s) = C(s)\hat{r}(s) + K_g r(s), \quad (8)$$

where $C(s)$ is a low-pass filter with low-pass gain 1, i.e. $C(0) = 1$,

$$K_g = (-C_p^\top A_m^{-1} B_p)^{-1}, \quad (9)$$

while $\hat{r}(s)$ is the Laplace transformation of the signal $\hat{r}(t) = \hat{W}^\top(t)\Phi(x(t))$, $r(s)$ is the Laplace transformation of $r(t)$, and $u_{ad}(s)$ is the Laplace transformation of $u_{ad}(t)$.

Consider the closed loop companion model with the control signal defined in (8). It can be viewed as an linear time-invariant (LTI) system with two inputs $r(t)$ and $\hat{r}(t)$:

$$\begin{aligned}\hat{x}(s) &= \hat{G}(s)\hat{r}(s) + G(s)r(s) \\ \hat{G}(s) &= H_o(s)(C(s) - 1) \\ G(s) &= H_o(s)K_g,\end{aligned}$$

where the stable transfer function $H_o(s)$ is given by

$$H_o(s) = (sI - A_m)^{-1}B_p,$$

and $\hat{x}(s)$ is the Laplace transformation of $\hat{x}(t)$. To ensure boundedness of the companion system and desired transient performance for the controller in (8), K and $C(s)$ need to be selected in a way to satisfy

$$\|\hat{G}(s)\|_{\mathcal{L}_1} < \frac{1}{L}, \quad (10)$$

where $\|\cdot\|_{\mathcal{L}_1}$ denotes the $\mathcal{L}_1$ gain of $\hat{G}(s)$, defined as

$$\|\hat{G}(s)\|_{\mathcal{L}_1} = \max_{i=1,\dots,n} \left(\sum_{j=1}^m \|\hat{G}_{ij}(s)\|_{\mathcal{L}_1} \right),$$

in which $\hat{G}_{ij}(s)$ is the i^{th} row j^{th} column element of $\hat{G}(s)$ (for details refer to [2]–[4]). We recall that the $\mathcal{L}_1$ gain of a stable proper single-input single-output system $H(s)$ is defined as $\|H\|_{\mathcal{L}_1} = \int_0^\infty |h(t)|dt$, where $h(t)$ is the impulse response of $H(s)$.

Following the approach in [2], let

$$\mathcal{D}_x = \{x \mid \|x\|_\infty \leq \gamma_r + \gamma_1 + \gamma_0 + \sigma\}, \quad (11)$$

where $\sigma > 0$ is an arbitrary positive constant, while

$$\gamma_r = \frac{\|G(s)\|_{\mathcal{L}_1} \|r\|_{\mathcal{L}_\infty} + \|\bar{G}(s)\|_{\mathcal{L}_1} \bar{B} + \|H_o(s)\|_{\mathcal{L}_1} \epsilon^*}{1 - \|\bar{G}(s)\|_{\mathcal{L}_1} L}, \quad (12)$$

$$\gamma_0 = \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} \left(\frac{2\epsilon^* \|PB_p\|}{\lambda_{\min}(Q)} \right)^2 + \frac{W_{\max}}{\lambda_{\min}(P)\Gamma_c}}, \quad (13)$$

$$\begin{aligned}\gamma_1 &= \frac{(5\|\bar{G}(s)\|_{\mathcal{L}_1} + \|H_o(s)\|_{\mathcal{L}_1})\epsilon^*}{1 - \|\bar{G}(s)\|_{\mathcal{L}_1} L} \\ &\quad + \frac{(1 + \|C(s) - 1\|_{\mathcal{L}_1})\gamma_0}{1 - \|\bar{G}(s)\|_{\mathcal{L}_1} L},\end{aligned}\quad (14)$$

where $\bar{B} = B + \epsilon^*$, $W_{\max} \triangleq 8 \max_{W_i \in \Theta} \|W_i\|^2$. The complete $\mathcal{L}_1$ adaptive controller consists of (3), (6), (7), (8) subject to (10) with $\mathcal{D}_x$ defined in (11).

IV. ANALYSIS OF THE $\mathcal{L}_1$ ADAPTIVE CONTROLLER

We need to characterize the reference system, which is being tracked by both the system state and the control input of the system (2) via the $\mathcal{L}_1$ adaptive controller both in transient and steady state. Following the approach in [2]–[4], consider the ideal version of the adaptive controller in (3) and (8):

$$u_{ref}(s) = K_g r(s) + \eta(s) - K^\top x_{ref}(s), \quad (15)$$

where $\eta(s)$ is the filtered output of $W^\top \Phi(x_{ref}(t))$ by $C(s)$, $x_{ref}(s)$ is the Laplace transformation of the state $x_{ref}(t)$ of the closed loop system. The closed-loop system with the ideal controller takes the form:

$$\begin{aligned}x_{ref}(s) &= G(s)r(s) + \hat{G}(s)\eta_1(s) - H_o(s)\epsilon(s) \\ y_{ref}(s) &= C_p^\top x_{ref}(s),\end{aligned}\quad (16)$$

where $\epsilon(s)$ and $\eta_1(s)$ are the Laplace transformation of the signals $\epsilon(x_{ref}(t))$ and $W^\top \Phi(x_{ref}(t))$, and $x_{ref}(0) = x_0$. The following lemma states that the closed-loop system with controller (15) is stable and its state remains inside $\mathcal{D}_x$ for all $t \geq 0$.

Lemma 1: [2] The control signal given by (15), subject to the condition in (10) ensures that the state of the closed-loop system in (16) remains inside $\mathcal{D}_x$ for all $t \geq 0$:

$$\|x_{ref}\|_{\mathcal{L}_\infty} \leq \gamma_r. \quad (17)$$

Thus the control signal ensures that for any $t \geq 0$ the state $x_{ref}(t) \in \mathcal{D}_x$ on which the RBF approximation has been defined.

Next we need to show the uniform boundedness and guaranteed transient performance of $\mathcal{L}_1$ neural network

adaptive controller. The main result from [2]–[4] is given by the following theorem.

Theorem 1: Given the system in (4), reference system in (15), (16), and the $\mathcal{L}_1$ neural network adaptive controller defined via (3), (6) (7), (8) subject to (10), we have:

$$\|x - x_{ref}\|_{\mathcal{L}_\infty} \leq \gamma_1, \quad (18)$$

$$\|y - y_{ref}\|_{\mathcal{L}_\infty} \leq \|C_p^\top\|_{\mathcal{L}_1} \gamma_1, \quad (19)$$

$$\|u - u_{ref}\|_{\mathcal{L}_\infty} \leq \gamma_2, \quad (20)$$

where $\|C_p^\top\|_{\mathcal{L}_1}$ is the $\mathcal{L}_1$ gain of $C_p^\top$, and

$$\begin{aligned} \gamma_2 = & \|C(s)(c_o^\top H_o(s))^{-1} c_o^\top\|_{\mathcal{L}_1} \gamma_0 \\ & + (\|C(s)\|_{\mathcal{L}_1} L + \|K^\top\|_{\mathcal{L}_1}) \gamma_1 + 3\|C(s)\|_{\mathcal{L}_1} \epsilon^*, \end{aligned} \quad (21)$$

in which $c_o \in \mathbb{R}^{6 \times 2}$ is a matrix that renders $c_o^\top H_o(s)$ minimum phase with relative degree 1 in each channel¹, and $\|K^\top\|_{\mathcal{L}_1}$ is the $\mathcal{L}_1$ gain of $K^\top$.

From the relationship in (17) and (18) it is straightforward to verify that $\|x\|_{\mathcal{L}_\infty} \leq \gamma_r + \gamma_1$ for any $t \geq 0$, i.e. $x(t) \in \mathcal{D}_x$.

Corollary 1: For the system in (4) and the $\mathcal{L}_1$ adaptive controller defined via (3), (6), (7), (8) subject to (10), we have:

$$\lim_{\Gamma_c \rightarrow \infty, \epsilon^* \rightarrow 0} (x(t) - x_{ref}(t)) = 0, \quad \forall t \geq 0, \quad (22)$$

$$\lim_{\Gamma_c \rightarrow \infty, \epsilon^* \rightarrow 0} (y(t) - y_{ref}(t)) = 0, \quad \forall t \geq 0, \quad (23)$$

$$\lim_{\Gamma_c \rightarrow \infty, \epsilon^* \rightarrow 0} (u(t) - u_{ref}(t)) = 0, \quad \forall t \geq 0. \quad (24)$$

Corollary 1 states that $x(t)$, $y(t)$ and $u(t)$ follow $x_{ref}(t)$, $y_{ref}(t)$ and $u_{ref}(t)$ not only asymptotically but also during the transient, provided that the adaptive gain is selected sufficiently large and the neural network approximation is accurate enough. Thus, the control objective is reduced to designing K and $C(s)$ to ensure that the reference system with unknown parameters has the desired response $D(s)$ from $r(t)$ to $y_{ref}(t)$. In [2]–[4], specific design guidelines are provided for selection of $C(s)$ to ensure that the system in (16) with (15) has the desired response $D(s)$ from $r(t)$ to $y_{ref}(t)$.

The following remarks are in order, [2]–[4].

Remark 1: In case if $C(s) = 1$, the $\mathcal{L}_1$ controller degenerates into a MRAC type. In that case, the term $\|C(s)(c_o^\top H_o(s))^{-1} c_o^\top\|_{\mathcal{L}_1}$ in γ_2 in (21) cannot be finite since $H_o(s)$ is strictly proper. Therefore, $\gamma_2 \rightarrow \infty$, which implies that in conventional MRAC type neural network adaptive controller one cannot reduce the bound of the control signal in (20) by increasing the adaptive gain or improving the approximation accuracy.

Remark 2: Recall that in conventional MRAC scheme the ultimate bound is given by γ_0 defined in (13). γ_0 depends upon ϵ^* , $W_{\max}$ and Γ_c . While ϵ^* and $W_{\max}$ are related via the choice of RBFs, Γ_c is a design parameter

¹Existence of such c_o can be proven using tools from linear systems theory [2]–[4].

of the adaptive process that can be used to reduce the ultimate bound. However, increasing the adaptive gain in conventional MRAC leads to high-frequency oscillations in the control signal. With the $\mathcal{L}_1$ adaptive control architecture the ultimate bound of the tracking error is given by γ_1 in (18). From the definition of it in (14) it follows $\gamma_1 > \gamma_0$. Nevertheless, the ability of the $\mathcal{L}_1$ control architecture to tolerate high adaptive gains implies that γ_0 can be reduced leading to an overall smaller value for γ_1 . This ability is enabled via the low-pass system in the feedback path that filters out the high-frequencies in $\hat{W}(t)\Phi(x(t))$ excited by large Γ_c .

V. SIMULATIONS

A simplified longitudinal model of F-16 aircraft is used in the simulation. The aircraft parameters are taken from [1], while the vortex data from the tanker are the same as in [17]. The tanker aircraft is in steady level flight. The target point for the receiver aircraft is chosen to be the center of the outer cross section of the drogue. The aircraft is trimmed at $V_0 = 502 \text{ ft/sec}$, $\alpha_0 = 0.03691 \text{ rad}$ and $\theta_0 = 0.03691 \text{ rad}$ at the altitude of $h = 20000 \text{ ft}$. The radius of the drogue is $r_d = 1.8 \text{ ft}$.

The linear control part $u_{lin}(t) = -K^\top x(t)$ is designed to achieve the desired $A_m = A_p - B_p K^\top$, where A_m is the system matrix of the state space representation of the desired LTI system $D(s)$. We place the poles of $D(s)$ at $(-0.03, -0.5, -0.05, -0.4, -0.6+6i, -0.6-6i)$. This leads to the following

$$K^\top = \begin{bmatrix} 0.02 & 0.5 & 0.002 & 0.003 & 38. & 2 \\ 0.0001 & 0.002 & 0.001 & 0.034 & 36 & 0.7 \end{bmatrix}$$

and

$$\begin{aligned} D_{11}(s) &= \frac{s^4 + 1.7s^3 + 37s^2 + 19s + 0.5}{s^6 + 2s^5 + 38s^4 + 36s^3 + 10s^2 + 0.6s + 0.01} \\ D_{12}(s) &= \frac{-2s^3 - 40s^2 - 35s - 1}{s^6 + 2s^5 + 38s^4 + 36s^3 + 10s^2 + 0.6s + 0.01} \\ D_{21}(s) &= \frac{-1.1s - 0.056}{s^6 + 2s^5 + 38s^4 + 36s^3 + 10s^2 + 0.6s + 0.01} \\ D_{22}(s) &= \frac{512s^2 + 236s + 11}{s^6 + 2s^5 + 38s^4 + 36s^3 + 10s^2 + 0.6s + 0.01}, \end{aligned}$$

where $D_{ij}(s)$ denotes the transfer function from the j^{th} input to the i^{th} output.

For the $\mathcal{L}_1$ controller design, we choose a third order low pass filter $C(s) = \frac{3\omega^2 s + \omega^3}{(s + \omega)^3}$, where ω is the bandwidth of it. 125 RBFs have been selected for the implementation of the adaptive controller. A conservative Lipschitz constant for the uncertainty is computed from the experimental data to be $L = 0.17$. For this $C(s)$, one can compute $\|\hat{G}(s)\|_{\mathcal{L}_1}$ numerically as a function of ω . In Fig. 1, we plot $\|\hat{G}(s)\|_{\mathcal{L}_1}$ with respect to ω , and compare it with $\frac{1}{L} = 5.8824$. We observe that setting $\omega > 5$ verifies the condition in (10). So we choose $\omega = 6$, and we set $\Gamma_c = 1000$.

Case 1. We apply the $\mathcal{L}_1$ controller to three different cases of initial conditions (small, intermediate

Fig. 1. $\|\hat{G}(s)\|_{\mathcal{L}_1}$ (solid) and $\frac{1}{L}$ (dotted) w.r.t. ω

Fig. 2. Performance of $\mathcal{L}_1$ controller for different initial conditions

and large), by which we mean that the refueling aircraft starts the maneuver from different initial distances as compared to the tanker. By “small” we mean $x_0 = [-300 \ 0 \ 300 \ 0 \ 0 \ 0]^T$, which corresponds to the case, when the receiver is $300ft$ behind the tanker, $300ft$ below the tanker at the beginning of the maneuver. “Intermediate” corresponds to $x_0 = [-500 \ 0 \ 500 \ 0 \ 0 \ 0]^T$, and “large” is the case, when $x_0 = [-800 \ 0 \ 800 \ 0 \ 0 \ 0]^T$.

Results are displayed in Figs. 2, 3. Fig. 4 presents the zoomed version of the control effort at the beginning of the maneuver. The oscillations correspond to the time instant when the reference command is initialized (1 sec upon the start of the maneuver). Fig 3 shows low-frequency oscillations in the control signals upon 80 seconds. This is the time when the receiver aircraft enters the vortex area of the tanker.

For all three initial distances, the same $\mathcal{L}_1$ adaptive controller is applied without any re-tuning of the parameters. We observe that for scaled initial conditions the input/output signals of the uncertain receiver aircraft have scaled responses, similar to linear systems. This verifies

Fig. 3. Time history of $u(t)$ for different initial conditions

Fig. 4. Time history of $u(t)$ for different initial conditions - at the beginning

the fact that the $\mathcal{L}_1$ controller approximates desired linear system's response $D(s)$ both in transient and steady state. We notice that starting the maneuver from different initial conditions the time to contact the drogue also scales.

Case 2. Next, we try to enforce uniform time interval from different initial conditions for the refueling aircraft to contact the drogue. For a specific initial condition $x_0 = [-500 \ 0 \ 500 \ 0 \ 0 \ 0]^T$ and for the drogue radius $r_d = 1.8ft$ we prescribe a finite time $T_f = 200sec$ for the refueling aircraft to capture the drogue. With the same $\mathcal{L}_1$ controller of **Case 1**, at time $t' = 200sec$ we have $\|y(t') - r(t')\|_2 = 1.69 < 1.8ft$.

If we change the initial condition to a larger position $x_0 = [-1000 \ 0 \ 1000 \ 0 \ 0 \ 0]^T$, then the above controller with the same $D_{ij}(s)$ will not send the receiver aircraft to the desired position within 200sec. If we want to enforce the same time for the refueling aircraft to contact the drogue from this initial condition, we need to choose relatively fast

Fig. 5. $\|\hat{G}(s)\|_{\mathcal{L}_1}$ (solid) and $\frac{1}{L}$ (dotted) w.r.t. ω

Fig. 6. Time history of $y(t)$ (solid) and $y_D(t)$ (dotted), i.e. $x_0 = [-1000 \ 0 \ 1000 \ 0 \ 0 \ 0]^T$

reference systems

$$\begin{aligned} D_{11}(s) &= \frac{s^4 + 1.6s^3 + 21s^2 + 13s + 0.6}{s^6 + 2.2s^5 + 22s^4 + 25s^3 + 8.7s^2 + 0.7s + 0.02} \\ D_{12}(s) &= \frac{-3.8s^3 - 55s^2 - 49s - 2}{s^6 + 2.2s^5 + 22s^4 + 25s^3 + 8.7s^2 + 0.7s + 0.02} \\ D_{21}(s) &= \frac{-0.63s - 0.03}{s^6 + 2.2s^5 + 22s^4 + 25s^3 + 8.7s^2 + 0.7s + 0.02} \\ D_{22}(s) &= \frac{512s^2 + 293s + 15.7}{s^6 + 2.2s^5 + 22s^4 + 25s^3 + 8.7s^2 + 0.7s + 0.02}, \end{aligned}$$

which lead to $\|y_D(t') - r(t')\|_2 = 1.13ft < 1.8ft$ for $t' = 140$ sec, where y_D denotes the output of this reference system. For this system, $\|\hat{G}(s)\|_{\mathcal{L}_1}$ as a function of ω is given in Fig. 5. Setting $\omega = 7$ verifies the condition in (10). We use the same adaptive gain as before, $\Gamma_c = 1000$. Fig. 6 shows the tracking performance of the $\mathcal{L}_1$ adaptive controller. For $t \geq t' = 140$ sec we observe that $\|y(t') - r(t')\|_2 = 1.14 < 1.8ft$.

Thus, to start the aerial refueling maneuver from different initial conditions and to guarantee the same uniform time for the probe to capture the drogue, with the $\mathcal{L}_1$ adaptive controller in the feedback loop the design is reduced to systematic selection of the desired linear reference system, which can be done using tools from linear systems theory,

as opposed to ad-hoc tuning of the adaptive gain, which is typical for MRAC controllers.

VI. CONCLUSION

Novel $\mathcal{L}_1$ neural network based adaptive control design method is presented to solve the autonomous aerial refueling problem in highly uncertain dynamic environment. The method achieves guaranteed transient performance for system's both signals, inputs and outputs, simultaneously, via systematic choice of design parameters. The control signals remain in low-frequency range.

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