Motion Estimation via a Zoom Camera

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This paper discusses the motion estimation problem of a rigid-body using a zoom camera via observations of a single feature point. Both the rotational and translational parameters are assumed to be unknown constants. Estimation of the unknown parameters is obtained in two steps. In the first step, the camera's focal length is fixed and the rotational parameters can be estimated. In the second step, the camera's focal length is changing as a time-varying signal. In our current design, the focal length performs a sinusoidal motion. The unknown depth, along with the translational parameters, can be estimated. Simulation results are presented that demonstrate successful motion estimation.

Key Words: Motion estimation, depth from zooming.

I. Introduction

The problem of 3D motion estimation has been a central topic in development of vision-based navigation and control algorithms. Solutions to this problem can perceptibly affect applications such as vision-based target tracking, coordination, navigation and obstacle avoidance. Various solutions to this problem have been proposed, such as nonlinear optimization techniques¹⁻³, linear least-square methods⁴, and extended Kalman filter^{5,6}, in addition to a class of nonlinear observers that arise from a control view point. These nonlinear observers, typically named as perspective nonlinear observers, study the perspective dynamic systems that result from the perspective projection. In general, a perspective dynamic system is a linear system whose output is observed up to a homogeneous line. It is due to this perspective projection that depth ambiguity occurs.

The problem of 3D motion estimation via perspective nonlinear observers has evolved through two stages in its development. First, range identification was addressed, where all the motion parameters of the moving object have been assumed to be known. The objective was to estimate the depth, or, otherwise saying, the range. A variety of nonlinear observers have been proposed, including the identifier-based observer (IBO)⁷, the sliding mode-based observer⁸, the Lyapunov-based method⁹, the linear approximation-based observer¹⁰, and the reduced-order observer¹¹, to name a few. Next, 3D motion estimation was considered that estimates both the unknown depth/position and all (or some) of the unknown motion parameters by employing stereo vision¹², running multiple cameras simultaneously¹³, integrating vision with range sensors¹⁴, active vision (moving the camera on a rod)¹⁵, or by modifying the behavior of the follower on which the camera

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is installed to provide persistent excitation to recover the target's information¹⁶. Alternatively, in this paper, we present a solution to the 3D motion estimation problem by employing a single zoom camera.

Inferring 3D information from a set of images taken by a zoom camera has been discussed in Refs. 17–22. It was reported in Refs. 18, 20, 22 that the pinhole projection model is inadequate to model the zooming effect, and more complicated models, such as thin-lens and thick-lens models, have to be used. In the aforementioned depth recovery results, the distance between the camera and the scene (target) is assumed to be fixed. The problem addressed aims at recovering the relative distance by taking several (at least two) shots of the scene/target with different zooming scales. The fundamental method is based on either direct computation using two views^{17, 20, 22}, or least-square minimization using multiple views²¹. In this paper, we exploit a recursive approach where estimation of the 3D information is updated upon acquiring each new image. More specifically, we discuss the 3D motion estimation problem of a rigid body, by observing one single feature point on the object and assuming that the object is moving according to a linear rigid-body motion. We appeal to nonlinear observer(s) and design of a time-varying motion of the camera's adjustable focal length to recover all the unknown motion parameters and the (absolute) 3D position of the target.

We show that 3D motion estimation of a rigid-body motion can be achieved via a zoom camera through two separate steps. In the first step, the zoom camera is used as a fixed focal-length camera. The three rotational parameters of the rigid-body motion can be estimated in this step, similar to the method presented in Ref. 12. The zoom camera's focal length performs a time-varying motion in the second step to enable estimation of the unknown depth and the translational parameters by providing persistent excitation, and thus eliminating the need for additional information or extra sensors. Therefore, by employing the use of an intrinsic parameter unique to the mechanical active vision system, i.e., the adjustable focal length (also known as zoom), depth recovery and motion estimation of a rigid-body motion can be achieved without modifying the vehicles' behavior or utilizing more sensors. Apparently, utilizing a zoom camera avoids the requirement of an additional camera as in the stereo vision setup. The image processing task can therefore be simplified.

Our proposed method is feasible due to two factors. First, it is assumed that the motion parameters are unknown constants. This renders the resulting perspective dynamic systems into linearly parameterized structure so that existing perspective nonlinear observers can be applied. These perspective nonlinear observers include those proposed in Refs. 7–9, 11, 12, 23–27, to name but a few. In this paper, the IBO described in Refs. 7, 28, 29 is used for state and parameter estimation. Next, the depth ambiguity is resolved by implementing adjustable focal length.

The work described in this paper stems from a project on vision-based target tracking of a ground vehicle for a small unmanned aerial vehicle equipped with a gimbaled Pan-Tilt-Zoom camera^{30–33}. This paper is organized as follows. Section II briefly reviews several projection models for a zoom camera where the thin-lens model is utilized in this paper for the purpose of 3D motion estimation. Detailed procedures of our proposed method are described in Sec. III. Simulation results are given in Sec. IV. Finally, Section V concludes the paper.

II. Projection Models for a Zoom Camera

The degrees of freedom in active vision systems include both extrinsic parameters (similar to eye movements) and intrinsic parameters (eye configuration). We are interested in exploring the use

of the adjustable focal length (also known as zoom), an intrinsic parameter unique to mechanical active vision. To explain the optical phenomenon occurring during a zoom-lens focal-length change, it has been observed that the simplest pinhole model is inadequate and more complicated models must be considered ^{18,20,22}.

There are four camera projection models currently in use ^{17,20,34}:

1. **Pinhole Model:** The pin-hole model assumes that all light rays coming from the object focus through a single point (the focal point) onto the image plane. This geometrically simple model is widely used in computer vision. However, it is only accurate if the focal length, i.e., the distance between the image plane and the focal point, is negligible compared to the object distance (e.g., in aerial photography) ²⁰.
2. **Thin-Lens Model:** The thin-lens model assumes an infinitely thin lens, modelled by a plane. Light rays from the object hitting the plane at a particular angle continue after leaving the plane at a modified angle ¹⁷.
3. **Thick-Lens Model:** The thick-lens model represents a lens by two planes called the principle planes. Light rays entering one plane at a specific angle travel parallel to the optical axis through to the second plane, from which they exit at another angle (similar to the thin-lens model). Ignoring aperture effects, this model is equivalent to the pinhole model with the addition of a virtual axial motion, and it is sufficient for modeling zoom lenses, even at close range ^{20,34}.
4. **Geometric-Lens Model:** The geometric-lens model is based on a complete geometric description of all of the glass elements of the lens. It is the most detailed and accurate of the models, and can be used to design lenses and to model effects such as optical aberrations, lens flare, etc. However, this model is extremely complicated to implement ²⁰.

Let $[X, Y, Z]^T$ be the coordinates of a point in the camera frame. Projections on the image plane when using the thin-lens and the thick-lens models are summarized in the following:

1. **Thin-Lens Model** ¹⁷:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{-f}{Z - f} \begin{bmatrix} X \\ Y \end{bmatrix}, \quad (1)$$

where f denotes the camera's focal length.

2. **Thick-Lens Model** ^{20,34}:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{f}{Z + l - f} \begin{bmatrix} X \\ Y \end{bmatrix}, \quad (2)$$

where f , the effective focal length of the zoom lens, is related to l by

$$\frac{1}{f} = \frac{1}{f_a} + \frac{1}{f_b} - \frac{l}{f_a f_b}, \quad (3)$$

where f_a and f_b are the focal lengths of the two lens groups corresponding to the two principal planes determined during calibration. Zoom camera calibration techniques can be found in Refs. ^{19,35–38}.

Table 1. Projection Models for a Zoom Camera

Model	Graphic Illustration	Projection
Pinhole		$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{f}{Z} \begin{bmatrix} X \\ Y \end{bmatrix}$ $f \text{ known}$
Thin-Lens		$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{-f}{Z - f} \begin{bmatrix} X \\ Y \end{bmatrix}$ $f \text{ known}$
Thick-Lens		$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{f}{Z + l - f} \begin{bmatrix} X \\ Y \end{bmatrix}$ <p>f and l are related by</p> $\frac{1}{f} = \frac{1}{f_a} + \frac{1}{f_b} - \frac{l}{f_a f_b},$ <p>where f_a and f_b are known.</p>

The pinhole, thin-lens and thick-lens projection models are illustrated in Table 1.

Both thin and thick-lens models have the same capability to represent the optical properties if the lens thickness has no physical meaning²¹. For our purpose, the thin-lens and thick-lens models are fundamentally similar from the perspective of the mathematical derivation of the estimation problem. The thin-lens model (1) is used in the following derivation. Similar procedure can be performed for the thick-lens model. However, this is not elaborated further in this paper.

III. Motion Estimation

Consider the movement of the rigid object described by^{7,9}:

$$\begin{bmatrix} \dot{X}(t) \\ \dot{Y}(t) \\ \dot{Z}(t) \end{bmatrix} = \begin{bmatrix} 0 & \omega_1 & \omega_2 \\ -\omega_1 & 0 & \omega_3 \\ -\omega_2 & -\omega_3 & 0 \end{bmatrix} \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, \quad (4)$$

where $[X(t), Y(t), Z(t)]^\top$ denotes the position of the feature point in the space; the matrix $[\omega_i]$ presents the rotational dynamics, and the vector $[b_i]$ corresponds to the translational motion. The observed position on the image plane of a zoom camera is defined by

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \frac{1}{Z(t) - f(t)} \begin{bmatrix} X(t) \\ Y(t) \end{bmatrix}, \quad (5)$$

where the thin-lens model in (1) is used and $f(t)$ is the time-varying focal length.

Let

$$\begin{aligned}\boldsymbol{\omega} &= [\omega_1, \omega_2, \omega_3]^\top, \\ \boldsymbol{b} &= [b_1, b_2, b_3]^\top.\end{aligned}\tag{6}$$

We make the following assumptions:

1. The motion parameters $\boldsymbol{\omega}$ and $\boldsymbol{b}$ are bounded constants.
2. $Z(t)$ meets the condition $Z(t) > \epsilon > 0$ for all $t \geq 0$, where ϵ is a constant.
3. $x_1(t)$ and $x_2(t)$ are bounded for all $t \geq 0$.

Our objective is to estimate $[X(t), Y(t), Z(t)]^\top$ and the motion parameters $\boldsymbol{\omega}$ and $\boldsymbol{b}$ by using the perspective observations of a single feature point, namely $x_1(t)$ and $x_2(t)$ as given in (5).

Let

$$x_3(t) = \frac{1}{Z(t) - f(t)}.\tag{7}$$

The resulting perspective dynamic system, obtained by applying equations (5) and (7) to (4), is given by:

$$\begin{cases} \dot{x}_1(t) = \omega_1 x_2(t) + \omega_2 (1 + x_1^2(t)) + \omega_3 x_1(t) x_2(t) + b_1 x_3(t) - x_1 b_3 x_3(t) + f(t) \omega_2 x_3(t) + \dot{f}(t) x_1(t) x_3(t), \\ \dot{x}_2(t) = -\omega_1 x_1(t) + \omega_2 x_1(t) x_2(t) + \omega_3 (1 + x_2^2(t)) + b_2 x_3(t) - x_2(t) b_3 x_3(t) + f(t) \omega_3 x_3(t) + \dot{f}(t) x_2(t) x_3(t), \\ \dot{x}_3(t) = (\omega_2 x_1(t) + \omega_3 x_2(t)) x_3(t) - (b_3 - \dot{f}(t)) x_3^2(t). \end{cases}\tag{8}$$

It is obvious that the position of the object in the 3D space can be calculated as

$$X(t) = \frac{x_1(t)}{x_3(t)}, \quad Y(t) = \frac{x_2(t)}{x_3(t)}, \quad Z(t) = \frac{1}{x_3(t)} + f(t),\tag{9}$$

if $x_3(t)$ is available. We show that $\boldsymbol{\omega}$, $x_3(t)$ and $\boldsymbol{b}$ can be estimated via the following steps. Note that $\dot{f}(t)$ is a design signal and is therefore available.

III.A. Estimation of Rotational Parameters $\boldsymbol{\omega}$

Let

$$\boldsymbol{\theta}(t) = \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \\ \theta_3(t) \\ \theta_4(t) \\ \theta_5(t) \\ \theta_6(t) \end{bmatrix} \triangleq \begin{bmatrix} (b_1 + f\omega_2) x_3(t) \\ (b_2 + f\omega_3) x_3(t) \\ b_3 x_3(t) \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}.\tag{10}$$

By choosing a constant focal length such that

$$\dot{f}(t) = 0,$$

the motion dynamics in (8) take the following form:

$$\left\{ \begin{array}{l} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & -x_1 & x_2 & 1+x_1^2 & x_1x_2 \\ 0 & 1 & -x_2 & -x_1 & x_1x_2 & 1+x_2^2 \end{bmatrix}}_{w_\theta^\top(x_1(t), x_2(t))} \boldsymbol{\theta}(t), \\ \dot{\boldsymbol{\theta}}(t) = \begin{bmatrix} (\theta_5x_1 + \theta_6x_2 - \theta_3) \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \\ 0 \\ 0 \\ 0 \end{bmatrix}. \end{array} \right. \quad (11)$$

The system (11) exhibits the structure given in equation (23) in the Appendix, to which IBO may be applied by treating $\mathbf{x}_1(t) = [x_1(t), x_2(t)]^\top$, $\mathbf{x}_2(t) = \boldsymbol{\theta}(t)$, and $\phi(\mathbf{x}_1, \mathbf{u}) = 0$. The IBO is briefly reviewed in the Appendix. To apply the IBO, we need the following assumption for the system (11):

Assumption III.1 *There do not exist constants κ_i (for $i = 1, \dots, 6$) with $\sum_{i=1}^6 \kappa_i^2 \neq 0$ such that*

$$\sum_{i=1}^6 \kappa_i v_i(\tau) = 0, \quad (12)$$

for all $\tau \in [t, t + \mu]$, where $v_i(\tau)$ stands for the i^{th} column in $w_\theta^\top(x_1(t), x_2(t))$ in (11) and μ is a small positive constant.

Let $\hat{\boldsymbol{\theta}}(t)$ denote the estimate of $\boldsymbol{\theta}(t)$. The following observer can be designed for the system in (11):

$$\left\{ \begin{array}{l} \begin{bmatrix} \dot{\hat{x}}_1(t) \\ \dot{\hat{x}}_2(t) \end{bmatrix} = GA \begin{bmatrix} \hat{x}_1(t) - x_1(t) \\ \hat{x}_2(t) - x_2(t) \end{bmatrix} + w_\theta^\top(x_1(t), x_2(t)) \hat{\boldsymbol{\theta}}(t), \\ \dot{\hat{\boldsymbol{\theta}}}(t) = -G^2 w_\theta(x_1(t), x_2(t)) P \begin{bmatrix} \hat{x}_1(t) - x_1(t) \\ \hat{x}_2(t) - x_2(t) \end{bmatrix} + \begin{bmatrix} (\hat{\theta}_5x_1 + \hat{\theta}_6x_2 - \hat{\theta}_3) \begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \\ \hat{\theta}_3 \end{bmatrix} \\ 0 \\ 0 \\ 0 \end{bmatrix}. \end{array} \right. \quad (13)$$

According to Theorem V.1, there exists a positive constant G_0 such that choosing $G > G_0$ ensures that the estimation errors

$$[\hat{x}_1(t) - x_1(t), \hat{x}_2(t) - x_2(t), \hat{\boldsymbol{\theta}}(t)^\top - \boldsymbol{\theta}^\top]^\top$$

converge to zero exponentially. This gives $\hat{\boldsymbol{\omega}}(t)$. Clearly,

$$\hat{\boldsymbol{\omega}}(t) = [\hat{\theta}_4(t), \hat{\theta}_5(t), \hat{\theta}_6(t)]^\top, \quad (14)$$

where $\hat{\theta}_i(t)$ (for $i = 4, 5, 6$) denotes the i^{th} element of $\hat{\boldsymbol{\theta}}(t)$. The signals $\hat{\boldsymbol{\omega}}(t)$ will be used later to estimate $x_3(t)$ and $\mathbf{b}$.

The IBO has been applied to the range identification problem assuming some or all of the motion parameters are known^{7,39}. In this paper, all the motion parameters are assumed to be unknown constants. The objective is to estimate the position and the motion parameters via observations of a single feature point using a zoom camera. The IBO's exponential convergence property makes this approach viable.

III.B. Estimation of Unknown Depth $x_3(t)$

Let

$$\boldsymbol{\vartheta}(t) = \begin{bmatrix} \vartheta_1(t) \\ \vartheta_2(t) \\ \vartheta_3(t) \\ \vartheta_4(t) \end{bmatrix} \triangleq \begin{bmatrix} b_1 x_3(t) \\ b_2 x_3(t) \\ b_3 x_3(t) \\ x_3(t) \end{bmatrix}. \quad (15)$$

The system dynamics in (8) can be rewritten as

$$\begin{cases} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} \omega_1 x_2 + \omega_2(1 + x_1^2) + \omega_3 x_1 x_2 \\ -\omega_1 x_1 + \omega_2 x_1 x_2 + \omega_3(1 + x_2^2) \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 & -x_1 & f(t)\omega_2 + \dot{f}(t)x_1 \\ 0 & 1 & -x_2 & f(t)\omega_3 + \dot{f}(t)x_2 \end{bmatrix}}_{w_{\boldsymbol{\vartheta}}^{\top}(x_1(t), x_2(t))} \boldsymbol{\vartheta}(t), \\ \dot{\boldsymbol{\vartheta}}(t) = (\omega_2 x_1 + \omega_3 x_2 - \vartheta_3 + \dot{f}(t)\vartheta_4) \boldsymbol{\vartheta}(t). \end{cases} \quad (16)$$

Replacing $\boldsymbol{\omega} = [\omega_1, \omega_2, \omega_3]^{\top}$ by $\hat{\boldsymbol{\omega}}(t)$, where $\hat{\boldsymbol{\omega}}(t)$ is obtained in (14), the system (16) also fits into the form to which IBO can be applied. Accordingly, an observer for $\boldsymbol{\vartheta}(t)$ can be constructed. Similarly, we need the following assumption:

Assumption III.2 *There do not exist constants κ_i (for $i = 1, \dots, 4$) with $\sum_{i=1}^4 \kappa_i^2 \neq 0$ such that*

$$\sum_{i=1}^4 \kappa_i v_i(\tau) = 0, \quad (17)$$

for all $\tau \in [t, t + \mu]$, where $v_i(\tau)$ stands for the i^{th} column in $w_{\boldsymbol{\vartheta}}^{\top}(x_1(t), x_2(t))$ in (16) and μ is a small positive constant.

To achieve the observability condition in (17), the camera's focal length is designed to perform the following sinusoidal motion:

$$f(t) = 0.5 \sin(2\pi t). \quad (18)$$

Detailed observer for the system in (16) is not given here (see Sec. III.A for a similar procedure). Let $\hat{\boldsymbol{\vartheta}}(t)$ denote the estimate of $\boldsymbol{\vartheta}(t)$. Clearly,

$$\hat{x}_3(t) = \hat{\vartheta}_4(t), \quad (19)$$

where $\hat{\vartheta}_4(t)$ is the 4th element of $\hat{\boldsymbol{\vartheta}}(t)$. The signal $\hat{x}_3(t)$, together with $\hat{\boldsymbol{\omega}}(t)$ that has been estimated in Sec. III.A, will be used later to estimate the translational parameters $\mathbf{b}$. Note that upon obtaining $\hat{x}_3(t)$, estimate of the 3D position $[X(t), Y(t), Z(t)]^{\top}$ can be computed directly from $\hat{x}_3(t)$ and the visual measurements $x_1(t)$ and $x_2(t)$.

III.C. Estimation of Translational Parameters $\mathbf{b}$

From Sec. III.B, the translational parameters $\mathbf{b}$ can be directly calculated by using the estimates of $(\mathbf{b}(t)x_3(t))$ and $x_3(t)$. However, this approach is not preferable when $x_3(t)$ is small and estimation errors are not negligible^{12,40}. Alternately, we can construct a dynamic observer for the parameters $\mathbf{b}$. Under the assumption that $\mathbf{b}$ are constants, the first two equations in (8) can be rewritten to give the following equations in the form of equation (23):

$$\begin{cases} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} \omega_1 x_2 + \omega_2(1 + x_1^2) + \omega_3 x_1 x_2 \\ -\omega_1 x_1 + \omega_2 x_1 x_2 + \omega_3(1 + x_2^2) \end{bmatrix} + \begin{bmatrix} f(t)\omega_2 + \dot{f}(t)x_1 \\ f(t)\omega_3 + \dot{f}(t)x_2 \end{bmatrix} x_3 + \begin{bmatrix} 1 & 0 & -x_1 \\ 0 & 1 & -x_2 \end{bmatrix} x_3 \mathbf{b}, \\ \dot{\mathbf{b}} = 0. \end{cases} \quad (20)$$

Replacing $\boldsymbol{\omega} = [\omega_1, \omega_2, \omega_3]^\top$ by $\hat{\boldsymbol{\omega}}(t)$ and $x_3(t)$ by $\hat{x}_3(t)$, where $\hat{\boldsymbol{\omega}}(t)$ and $\hat{x}_3(t)$ are obtained in (14) and (19), respectively, an observer for $\mathbf{b}$ can be constructed by applying IBO to (20). Note that if Assumption III.2 holds, the convergence condition for the observer of (20) is also satisfied.

In summary, complete 3D estimation of a rigid-body motion is given in equations (13), (19) and by constructing an observer to (20). Our proposed method is feasible due to the following factors:

1. The assumption that the motion parameters are unknown constants allows for transforming the resulting perspective dynamic system into a linearly parameterized structure, such that existing perspective nonlinear observers can be applied (see equations (11), (16) and (20)).
2. The depth ambiguity is resolved applying adjustable focal length.

IV. Simulation Results

Considering the system in (8), motion estimation of a rigid-body with constant rotational and translational parameters is performed for the following dynamical system:

$$\begin{bmatrix} \dot{X}(t) \\ \dot{Y}(t) \\ \dot{Z}(t) \end{bmatrix} = \begin{bmatrix} 0 & 5 & 0.6 \\ -5 & 0 & 0.7 \\ -0.6 & -0.7 & 0 \end{bmatrix} \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \\ -0.1 \end{bmatrix}, \quad (21)$$

$$(X_0, Y_0, Z_0) = (1, 1.5, 2.5), \quad x_0 = (X_0, Y_0, Z_0)/Z_0.$$

In simulations, the visual measurements are corrupted with 1% uniform noise and a step size of 0.02 second is used. The observers' initial states are chosen to be $\hat{\boldsymbol{\theta}}(0) = \boldsymbol{\theta}(0) + [0.8, -0.5, 0.7, 0.5, 0.5, 0.5]^\top$, $\hat{\boldsymbol{\vartheta}}(0) = \boldsymbol{\vartheta}(0) + [0.8, -0.8, 0.5, 1]^\top$, and $\hat{\mathbf{b}}(0) = [4, 3, 0.9]^\top$. The observer parameters are chosen to be $M = 15$, $A = \mathbb{I}_2$, $P = -1/2 \times \mathbb{I}_2$, and $G = 5$, where $\mathbb{I}_2$ denotes the 2×2 identity matrix. Without loss of generality, the camera's focal length f is fixed at 0.5 when constructing the observer (13). Later on, the focal length employs the sinusoidal motion in (18).

Estimations of $\boldsymbol{\omega}$, $x_3(t)$ and $\mathbf{b}$ are shown in Figures 1, 2, and 3, respectively, where true values are plotted in solid lines and corresponding estimates are in dashed. It can be seen that good estimates are obtained. Figure 4 shows the advantage of using the nonlinear observer to estimate $\mathbf{b}$ using direct computations by plotting the errors of these two methods for comparison.

Figure 1. Estimation of rotational parameters ω of (21).

Figure 2. Estimation of $x_3(t)$ of (21).

Figures 5-7 show simulation results of the following motion dynamics

$$\begin{bmatrix} \dot{X}(t) \\ \dot{Y}(t) \\ \dot{Z}(t) \end{bmatrix} = \begin{bmatrix} 0 & 4 & 0.5 \\ -4 & 0 & 0.7 \\ -0.5 & -0.7 & 0 \end{bmatrix} \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix} + \begin{bmatrix} 4 \\ 3 \\ -0.2 \end{bmatrix}, \quad (22)$$

$$(X_0, Y_0, Z_0) = (1, 1.5, 2.5), \quad x_0 = (X_0, Y_0, Z_0)/Z_0.$$

Similar to the simulations conducted on (21), the visual measurements are corrupted with 1% uniform noise and a step size of 0.02 second is used. The observers' initial states are chosen to be $\hat{\theta}(0) = \theta(0) + [0.8, -0.5, 0.7, 0.5, 0.5, 0.5]^\top$, $\hat{\vartheta}(0) = \vartheta(0) + [0.8, -0.8, 0.5, 1]^\top$, and $\hat{\mathbf{b}}(0) = [5, 4, 0.8]^\top$. It can be observed that both position estimation of $x_3(t)$ and parameter estimations of ω and $\mathbf{b}$ are obtained.

Figure 3. Estimation of translational parameters b of (21).

Figure 4. Comparison of estimation errors of the translational parameters between observer outputs (solid lines) and direct computations (dashed lines) of (21).

V. Conclusion

We present a solution to the estimation problem of a rigid-body motion using a zoom camera via observations of a single feature point. Assuming that all the motion parameters are unknown constants, these parameters, together with the unknown depth/range, can be estimated via two steps. In the first step, the zoom camera is used as a fixed focal-length camera. Similar to results available in the literature, the rotational parameters of the rigid-body motion can be estimated. The second step relies on the camera's adjustable focal length, which is required to perform a time-varying motion to provide persistent excitation for the estimation of the unknown range along with the translational parameters. A sinusoidal motion is currently adopted in this step.

By exploiting a unique intrinsic parameter to the active vision system, i.e., the adjustable focal length (also known as zoom), motion estimation of a rigid-body motion can be achieved without additional information or extra sensors. Having the estimation achieved in one unit (i.e., the zoom camera), the vehicle's behavior need not be modified. The image processing task can perceptibly be simplified as well, since the feature point correspondence problem is avoided.

Figure 5. Estimation of rotational parameters ω of (22).

Figure 6. Estimation of $x_3(t)$ of (22).

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Review of IBO

We review the IBO in this section. This observer is used for the estimation task. The convergence properties of IBO are proved for a class of nonlinear systems with the following structure ⁷:

$$\begin{cases} \dot{\mathbf{x}}_1(t) = w^\top(\mathbf{x}_1(t), \mathbf{u}(t))\mathbf{x}_2(t) + \phi(\mathbf{x}_1(t), \mathbf{u}(t)), \\ \dot{\mathbf{x}}_2(t) = g(\mathbf{x}_1(t), \mathbf{x}_2(t), \mathbf{u}(t)), \\ \mathbf{y}(t) = \mathbf{x}_1(t), \end{cases} \quad (23)$$

where $\mathbf{x}_1(t) \in \mathbb{X}_1 \subset \mathbb{R}^{n_1}$, $\mathbf{x}_2(t) \in \mathbb{X}_2 \subset \mathbb{R}^{n_2}$, and $\mathbf{u}(t) \in \mathbb{U} \subset \mathbb{R}^k$. The $n_1 \times n_2$ matrix $w^\top(\mathbf{x}_1(t), \mathbf{u}(t))$ and the vector $g(\mathbf{x}_1(t), \mathbf{x}_2(t), \mathbf{u}(t))$ are general nonlinear functions of their parameters. Let $\mathbf{x}(t) =$

Figure 7. Estimation of translational parameters $\mathbf{b}$ of (22).

$[\mathbf{x}_1^\top(t), \mathbf{x}_2^\top(t)]^\top$ and $n = n_1 + n_2$, so that $\mathbf{x}(t) \in \mathbb{X} \subset \mathbb{R}^n$, where $\mathbb{X} = \mathbb{X}_1 \oplus \mathbb{X}_2$.

Following Ref. 7, we introduce the following assumptions:

Assumption V.1

1. Let $\mathbf{x}(t)$ be bounded: $\|\mathbf{x}(t)\| < M$, where $M > 0$ for every $t \geq 0$. Let $\Omega = \{\mathbf{x} \in \mathbb{R}^n : \|\mathbf{x}(t)\| < M\}$. Further, for some fixed constant $\gamma > 1$, let $\Omega_\gamma = \{\mathbf{x}(t) \in \mathbb{R}^n : \|\mathbf{x}(t)\| < \gamma M\}$. Assume that the function $g(\mathbf{x}_1(t), \mathbf{x}_2(t), \mathbf{u}(t))$ is locally Lipschitz in Ω_γ with respect to $\mathbf{x}_2(t)$, i.e., there exists a positive constant α such that

$$\|g(\mathbf{x}_1, \mathbf{x}_2, \mathbf{u}) - g(\mathbf{x}_1, \mathbf{z}_2, \mathbf{u})\| < \alpha \|\mathbf{x}_2 - \mathbf{z}_2\|, \quad (24)$$

for all $\mathbf{x}_2(t), \mathbf{z}_2(t) \in \Omega_\gamma \cap \mathbb{X}_2$, uniformly in $\mathbf{x}_1(t) \in \Omega_\gamma \cap \mathbb{X}_1$ and $\mathbf{u}(t) \in \mathbb{U}$.

2. Let the regressor matrix $w^\top(\mathbf{x}_1(t), \mathbf{u}(t))$ and its first time derivative be piecewise smooth and uniformly bounded. Further, assume that there exist positive constants L_1, L_2, η , and μ such that

$$\|w^\top(\mathbf{x}_1, \mathbf{u})\| < L_1, \quad \left\| \frac{dw^\top(\mathbf{x}_1, \mathbf{u})}{dt} \right\| < L_2, \quad (25a)$$

$$\int_t^{t+\mu} w(\mathbf{x}_1(\tau), \mathbf{u}(\tau)) w^\top(\mathbf{x}_1(\tau), \mathbf{u}(\tau)) d\tau > \eta \mathbb{I} \quad (25b)$$

for all $t \geq 0$, for all trajectories that originate in $\mathbb{X}$ and for all $\mathbf{u}(t) \in \mathbb{U}$, where $\mathbb{I}$ denotes the identity matrix of appropriate dimension.

Letting $\hat{\mathbf{x}}_1(t)$ and $\hat{\mathbf{x}}_2(t)$ be the estimates of $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$, respectively, the IBO is introduced as:

$$\begin{cases} \dot{\hat{\mathbf{x}}}_1(t) = G\mathbf{A}(\hat{\mathbf{x}}_1 - \mathbf{x}_1) + w^\top(\mathbf{x}_1, \mathbf{u})\hat{\mathbf{x}}_2 + \phi(\mathbf{x}_1, \mathbf{u}), \\ \dot{\hat{\mathbf{x}}}_2(t) = -G^2 w(\mathbf{x}_1, \mathbf{u})P(\hat{\mathbf{x}}_1 - \mathbf{x}_1) + g(\mathbf{x}_1, \hat{\mathbf{x}}_2, \mathbf{u}), \\ \hat{\mathbf{x}}(t_i^+) = M \frac{\hat{\mathbf{x}}(t_i^-)}{\|\hat{\mathbf{x}}(t_i^-)\|}, \end{cases} \quad (26)$$

where G is a scalar constant and A is an $n_1 \times n_1$ Hurwitz matrix. The matrix P is the positive definite solution of the Lyapunov equation $A^\top P + PA = -Q$ for some $Q > 0$. The sequence t_i is defined as follows:

$$t_i = \min\{t : t > t_{i-1} \text{ and } \|\hat{\mathbf{x}}(t)\| \geq \gamma M\}, t_0 = 0, \quad (27)$$

where γ is a fixed constant.

Theorem V.1 ⁷ *Subject to Assumption V.1, there exists a positive constant G_0 such that the estimation errors $\mathbf{e}_1(t) = \mathbf{x}_1(t) - \hat{\mathbf{x}}_1(t)$ and $\mathbf{e}_2(t) = \mathbf{x}_2(t) - \hat{\mathbf{x}}_2(t)$ converge to zero exponentially if the constant G in (26) is chosen larger than G_0 .*

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