Active Control of Visual Sensor for Aerial Tracking*

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This paper addresses the problem of on-board estimation of the state and/or the velocity components of an unknown leader by open-loop control of a single camera installed on the follower aircraft. Unlike the conventional pan-tilt gimbal, a custom-built gimbal is assumed that permits translational movement of the camera. The only information about the unknown leader is deduced from its bearing and subtended angles in the image plane of the camera. An adaptive estimation algorithm is applied to estimate the state vector and the geometric parameters of the unknown object from visual measurements outside the flight control loop of the follower aircraft. Asymptotic convergence of the estimated parameters to the true ones is proven and shown to be independent of the flight controller. The estimated parameters can be used in the guidance law of the follower aircraft without any modifications to it. The main benefit of the proposed paradigm is in this decoupling of estimation from control.

I. Introduction

The main challenge in visual navigation and tracking is the unobservability aspect of the problem, which is inherent to visual sensors that project 3D objects onto a 2D plane, consequently rendering the range between the objects unobservable. For large scale airplanes, one can employ multiple cameras to recover the observability via stereo vision, but this is not practical in the development of small scale UAVs (Unmanned Aerial Vehicles). In this paper, we address the problem of active adaptive estimation in visual aerial tracking with the use of only one visual sensor. By active control of the visual sensor, the adaptive estimator provides the size, the range and the velocity information on moving or static objects. This estimation information can be utilized by the guidance law to meet its tracking objective directly without any modifications to it.

We notice that to recover observability in the problems of visual aerial tracking, Extended Kalman filter (EKF) or modified EKF have been the main tool for extracting necessary information about the dynamics of the moving objects.^{1,2,3,4,5} However, convergence properties for EKF have been proven only for limited classes of deterministic systems.⁶ Refs.^{7,8,9,10,11,12,13} have explored a variety of methods for visual tracking of moving objects. The main difference in these various approaches is in the class of maneuvers required from the follower to recover the observability.

In Ref.,¹⁴ these maneuvers have been characterized within an adaptive control framework as an intelligent excitation. Intelligent excitation ensures simultaneous parameter convergence (adaptive learning of the size of the leader) and output tracking, which leads to relative range recovery and facilitates the asymptotic tracking with visual sensor.

The special maneuvers in Refs.^{7,8,9,10,11,12,14,13} required for estimation and range recovery can disturb the normal operation of the aircraft, especially in the aerial-tracking operation mode. Unlike the above approaches which couple estimation and control, we propose a new approach in this paper that does not require special maneuvers for the aircraft to achieve parameter convergence or range recovery. In this paper we assume an “*active gimbal*”, which permits additional movement of the camera in addition to the 2-D freedom of rotation, provided by conventional pan-tilt gimbals. One way to achieve this is acentric rotation of the gimbal, or alternatively, one can let a gimbal move along a rod. Conventional 2-D freedom of the

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Figure 1. Comparison of different approaches.

rotational movement of the gimbal will keep the leader in the field of view, while the newly introduced translational movement will be controlled to achieve parameter convergence of the adaptive estimator. We prove that the estimation phase can be decoupled from feedback control.

Fig. 1 illustrates the difference in the implementation diagram of this new paradigm as compared to the approaches in Refs. ^{10,13,12,14} Fig. 1(a) shows a sketch for a conventional guidance law when the leader's state or geometric parameters are available. Fig. 1(b) demonstrates the modification to it, when the information on leader's state/parameters is extracted via adaptive estimation. We notice that it uses the same guidance law as in Fig. 1(a). The proposed design and implementation of the adaptive visual estimation algorithm in Fig. 1(b) do not require any modification to the guidance law. Similarly, the guidance law simply assumes that the adaptive visual estimation algorithm provides precise estimation results. Substituting the true values of the unknown parameters by their estimates does not affect the stability and asymptotic performance of the guidance law, as demonstrated in the proof in the sequel. We also note that the previous approaches to the problem described in Refs ^{10,13,12,14} relied on the architecture in Fig. 1(c) for the convergence of the estimation loop. The guidance law needed to be augmented by some type of excitation signal to ensure convergence of the adaptive estimation algorithm. We note that the estimation algorithm by itself could not guarantee parameter convergence without the modifications to the guidance law.^{12,14}

The paper is organized as follows. In Section II, we give the problem formulation and state the control objective. In Section III, we develop the active adaptive vision system and analyze its convergence. In Section IV, the guidance law for the follower UAV is presented that achieves the control objective. Simulation results are presented in Section V, and Section VI concludes the paper.

II. Problem Formulation

For the sake of simplicity, we consider the motion of two aircraft in the two-dimensional inertial Cartesian space (X, Y) (Figure 2) and denote positions of the leader and the follower in this inertial space as $P_L(t)$ and $P_F(t)$, respectively. Extension to 3D space is straightforward and is not pursued in this paper. Let $r(t) = [r_x(t) \ r_y(t)]^T = P_L(t) - P_F(t)$ be the vector of relative distance between the two vehicles in the inertial space. Let the lead aircraft maintain a constant velocity $V_L = [V_{L_x} \ V_{L_y}]^T$ in this inertial space, and let ϕ_L denote the angle of the vector V_L in (X, Y) space w.r.t. axis X . Similarly, we denote by $V_F(t)$ the velocity of the follower and by ϕ_F the angle of the vector V_F w.r.t. X axis in the inertial space (X, Y) . In addition, we denote by $\psi(t)$ the body orientation (yaw) angle of the follower w.r.t. X axis in inertial space

Figure 2. Bearing and Subtended Angles.

(X, Y) . The kinematics of the yaw angle can be described as:

$$\dot{\psi}(t) = V_{\psi}(t), \quad (1)$$

where $V_{\psi}(t)$ is the yaw rate of the follower. Consider the following transformation matrix:

$$T_f(\psi) = \begin{bmatrix} \sin(\psi) & \cos(\psi) \\ -\cos(\psi) & \sin(\psi) \end{bmatrix}, \quad (2)$$

and the transformation of the velocity vector

$$V_c(t) = T_f(\psi(t))V_F(t). \quad (3)$$

The follower has no knowledge of the lead vehicle, including its velocity V_L and characteristic dimension L , except maybe for some conservative upper and lower bounds on them. The only information about the leader can be drawn from a camera installed on the follower. The active vision system consists of a rod and a camera, or alternatively a gimbal system with a camera, on it. The rod is fixed on the UAV and aligned with one of the axis of its body frame (Figure 3). Without loss of generality, we assume that the rod is perpendicular to the longitudinal axis of the follower. The camera or the gimbal system can perform translational movement along this rod with commanded velocity $u_x(t)$. The follower measures its own states, which include $V_F(t)$, $\psi(t)$ and $V_{\psi}(t)$. The guidance law of the follower UAV needs to generate $V_{\psi}(t)$ and $V_c(t)$ to ensure that the follower can track the leader with desired prespecified separation.

We consider a coordinate system, (X_f, Y_f) , attached to the rod, which is fixed on the follower aircraft (Figures 2-3). Denote the coordinates of the vector $r(t)$ in (X_f, Y_f) space by $r_f(t) = [r_{fx} \ r_{fy}]^T$. It can be verified easily that

$$r_f(t) = T_f(\psi(t))r(t), \quad (4)$$

where $T_f(\psi)$ is defined via (2). Since the camera is moving along the rod, its velocity in the body frame (X_f, Y_f) is

$$[u_x(t) \ 0]^T.$$

We notice that $u_x(t)$ is a signal which can be open-loop controlled independent of the follower's motion. We can check easily that the camera's velocity and yaw angle in the inertial space (X, Y) are:

$$V_m(t) = u_c(t) + V_F(t),$$

Setup of Active Adaptive Vision Estimation System

Figure 3. Active camera system in the body frame.

where

$$u_c(t) = T_f^{-1}(\psi(t)) \begin{bmatrix} u_x(t) \\ 0 \end{bmatrix} = \begin{bmatrix} u_x \sin(\psi(t)) \\ -u_x \cos(\psi(t)) \end{bmatrix}.$$

As mentioned before, the only information about the leader's states and parameters can be deduced from the two angles as shown on Fig. 3. The first is the angle between the line of sight and the orientation of the camera, which is called bearing angle

$$\beta = \psi - \arctan(r_y/r_x), \quad (5)$$

while the second one is the subtended angle of the leader in the image plane

$$\alpha = 2 \arctan(L/(2\|r\|)). \quad (6)$$

With (5)-(6), the coordinates of relative motion can be expressed as:

$$r(\gamma) = \begin{bmatrix} \frac{L \cos(\psi - \beta)}{2 \tan(\alpha/2)} \\ \frac{L \sin(\psi - \beta)}{2 \tan(\alpha/2)} \end{bmatrix}. \quad (7)$$

In kinematic setting, the relative dynamics can be described via the following set of first order differential equations:

$$\dot{r}(t) = V_L(t) - V_m(t) = V_L(t) - V_F(t) - u_c(t). \quad (8)$$

It follows from (8) and (7) that

$$\frac{\partial r}{\partial \alpha} \dot{\alpha}(t) + \frac{\partial r}{\partial \beta} \dot{\beta}(t) + \frac{\partial r}{\partial \psi} \dot{\psi}(t) = V_L(t) - V_F(t) - u_c(t).$$

Let $\gamma(t) \triangleq (\alpha(t), \beta(t))$ be the vector of available visual measurements. Thus, the state space model of the relative motion of two vehicles can be written as:

$$\dot{\gamma}(t) = \frac{F_a(\gamma(t))}{L} (V_F(t) + u_c(t) - V_L(t)) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} V_\psi(t) \quad (9)$$

with $\gamma(0) = [\alpha_0 \ \beta_0]^\top$, where

$$F_a(\gamma) = \begin{bmatrix} 4 \sin^2(\alpha/2) \cos(\psi - \beta) & 4 \sin^2(\alpha/2) \sin(\psi - \beta) \\ -2 \tan(\alpha/2) \sin(\psi - \beta) & 2 \tan(\alpha/2) \cos(\psi - \beta) \end{bmatrix}, \quad (10)$$

Figure 4. Control objective in leader's body frame.

while $V_\psi(t) = \dot{\psi}(t)$ is the follower's yaw rate. We notice that from the definition of the subtended angle we have $0 < \alpha < \frac{\pi}{2}$. In addition, since $0 < \alpha(t) < \pi/2$ for all $t \geq 0$, we have

$$\det(F_a(\gamma(t))) = 8 \sin^2(\alpha/2) \tan(\alpha/2) \neq 0, \quad \forall t \geq 0. \quad (11)$$

Consider the body frame (X_l, Y_l) of the lead aircraft with its Y_l axis aligned with the direction of velocity V_L in the inertial space. We denote by $r_l(t) = [r_{lx} \ r_{ly}]^\top$ the coordinates of the vector $r(t)$ in (X_l, Y_l) . The objective is to design the open-loop control signal $u_x(t)$ for the active camera and the guidance laws $V_\psi(t)$ and $V_c(t)$ for the follower to ensure

$$\lim_{t \rightarrow \infty} \psi(t) = \phi_L, \quad (12)$$

$$\lim_{t \rightarrow \infty} \phi_F(t) = \phi_L, \quad (13)$$

$$\lim_{t \rightarrow \infty} r_l(t) = r_r. \quad (14)$$

The control objectives in (12)-(14) ensure that the follower flies in the same direction as the leader, and that in steady-state the follower maintains desired relative separation from the leader without any sideslip. We note that $\psi(t) \neq \phi_F(t)$ during the transient if the lateral velocity V_{c1} is not zero.

III. Aerial tracking using active vision system

We design two subsystems to achieve the visual tracking objective: an adaptive visual estimation algorithm and a guidance law for the follower UAV, Figure 5. The adaptive visual estimation algorithm provides $\hat{L}_e(t)$ and $\hat{V}_{le}(t)$ for the leader's characteristic length L and velocity $V_L(t)$, respectively, via the control of the active camera $u_x(t)$ and the adaptive estimator, which will be introduced shortly. The UAV's guidance law uses these estimates directly to achieve the control objective. We notice that the control of the active camera is independent of the aircraft's control. The guidance law is a conventional one and simply uses the signals provided by the estimation system. The two subsystems are completely decoupled from both design and implementation perspectives. In Section III.A, the active adaptive estimation system is introduced. The convergence of parameter estimation is shown in Section III.B. The guidance law is developed in Section IV.

Figure 5. Block diagram of overall estimation and control systems.

III.A. Active Adaptive Estimation System

Denoting the vector of unknown parameters as

$$\theta^* \triangleq [\theta_1^* \ \theta_2^* \ \theta_3^*]^\top = \left[\frac{1}{L} \ \frac{V_{Lx}}{L} \ \frac{V_{Ly}}{L} \right]^\top,$$

we assume that a conservative range for each of these unknown parameters is known

$$\theta_i^* \in \Theta_i \triangleq [\Theta_{i_{\min}}, \ \Theta_{i_{\max}}], \quad i = 1, 2, 3, \ \forall t \geq 0.$$

Hence, the system in (9) can be rewritten as:

$$\begin{aligned} \dot{\gamma}(t) &= F_a(\gamma(t)) F_c^\top(V_F(t), u_x(t)) \begin{bmatrix} \theta_1^* \\ \theta_2^* \\ \theta_3^* \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} V_\psi(t) \\ \gamma(0) &= \gamma_0 = [\alpha_0 \ \beta_0]^\top, \quad t \geq 0, \end{aligned} \tag{15}$$

where $F_a(\gamma(t))$ is defined in (10) and

$$F_c(V_F(t), u_x(t)) = \begin{bmatrix} V_{F_x}(t) + u_x(t) \sin(\psi(t)) & -V_{F_y}(t) - u_x(t) \cos(\psi(t)) \\ -1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{16}$$

Design of all the components of the active adaptive estimation system in Figure 5 is stated next.

III.A.1. Design of $u_x(t)$

We design the open-loop control signal of the active camera as:

$$u_x(t) = v \sin(\omega t), \quad (17)$$

where v and ω are positive design parameters. We note that ω needs to be selected in a way that $u_x(t)$ is linearly independent of $V_{F_x}(t)$ and $V_{F_y}(t)$ over any time interval $[t, t + \frac{2\pi}{\omega}]$. A simple choice is to choose ω larger than the possible oscillation frequencies in the follower's velocities. Since the active motion of the camera is limited by the physical constraints of the rod, the choice of the design gain v should satisfy:

$$2\frac{v}{\omega} < l, \quad (18)$$

where l is the length of the rod along which the camera performs translational motion.

III.A.2. Adaptive Estimator

For the system in (15), we consider the following adaptive estimator:

$$\begin{aligned} \dot{\hat{\gamma}}(t) &= A_m e(t) + F_a(\gamma(t)) F_c^\top(V_F(t), u_x(t)) \theta(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} V_\psi(t), \\ \hat{\gamma}(0) &= \gamma_0 = [\alpha_0 \ \beta_0]^\top, \quad t \geq 0, \end{aligned} \quad (19)$$

where A_m is given Hurwitz matrix, $F_a(\gamma(t))$, $F_c(V_{F_x}(t))$ are defined in (10), (16),

$$e(t) = \hat{\gamma}(t) - \gamma(t), \quad (20)$$

and $\theta(t) \triangleq [\theta_1(t) \ \theta_2(t) \ \theta_3(t)]^\top$ are estimated parameters. The adaptive laws for the parameter estimates are defined via the projection operator:¹⁵

$$\begin{aligned} \dot{\theta}(t) &= \Gamma \text{Proj}(-F_c(V_F(t), u_x(t)) F_a^\top(\gamma(t)) P e(t), \theta(t)), \\ \theta(0) &= \theta_0, \quad t \geq 0, \end{aligned} \quad (21)$$

where Γ is a positive definite matrix of adaptation gains, and $P = P^\top > 0$ solves the Lyapunov equation:

$$A_m^\top P + P A_m = -Q \quad (22)$$

for arbitrary $Q > 0$. Hence, the error dynamics can be derived easily:

$$\begin{aligned} \dot{e}(t) &= A_m e(t) + F_a(\gamma(t)) F_c^\top(V_{F_x}(t)) \tilde{\theta}(t), \\ \gamma(0) &= \gamma_0 = [\alpha_0 \ \beta_0]^\top, \quad t \geq 0, \end{aligned} \quad (23)$$

where

$$\tilde{\theta}(t) = [\tilde{\theta}_1(t) \ \tilde{\theta}_2(t) \ \tilde{\theta}_3(t)]^\top = \theta(t) - \theta^*.$$

The complete active adaptive estimator consists of (19)-(21) and produces $\theta(t)$ for estimation of the unknown parameters θ^* . Since $\theta(t)$ estimates θ^* , the estimates of L and V_L can be obtained from $\theta(t)$ as follows:

$$\begin{aligned} \hat{L}(t) &= 1/\theta_1(t), \\ \hat{V}_L(t) &= \frac{1}{\theta_1(t)} \begin{bmatrix} \theta_2(t) \\ \theta_3(t) \end{bmatrix}. \end{aligned} \quad (24)$$

III.A.3. Extraction via Low-pass Filters

Since the adaptive estimates $\hat{L}(t)$ and $\hat{V}_L(t)$ are not guaranteed to be free of high-frequency signals, we further low-pass filter these estimates to obtain low-frequency signals for estimation of the unknown parameters, i.e.

$$\begin{aligned}\hat{L}_e(s) &= C_1(s)\hat{L}(s), \\ \hat{V}_{le}(s) &= C_2(s)\hat{V}_L(s), \\ \hat{\phi}_L(t) &= \sin^{-1} \left(\frac{\hat{V}_{le}(t)}{\|\hat{V}_{le}\|} \right)\end{aligned}\tag{25}$$

where $\hat{L}_e(s)$, $\hat{V}_{le}(s)$, $\hat{L}(s)$, $\hat{V}_L(s)$ are the Laplace transformations of $\hat{L}_e(t)$, $\hat{V}_{le}(t)$, $\hat{L}(t)$, $\hat{V}_L(t)$, and $C_1(s)$, $C_2(s) = \begin{bmatrix} C_{21}(s) & 0 \\ 0 & C_{22}(s) \end{bmatrix}$ are stable low-pass filters with unity low-pass gains, i.e.

$$C_1(0) = C_{21}(0) = C_{22}(0) = 1.$$

III.B. Convergence Property of the Adaptive Estimator

We define a Lyapunov function candidate as:

$$\mathcal{V}(e(t), \tilde{\theta}(t)) = e^\top(t)Pe(t) + \tilde{\theta}^\top(t)\Gamma^{-1}\tilde{\theta}(t).\tag{26}$$

It can be verified easily that

$$\dot{\mathcal{V}}(t) \leq -e^\top(t)Qe(t),\tag{27}$$

where $Q < 0$ is defined in (22). Since θ^* is constant, application of Barbalat's lemma yields

$$\lim_{t \rightarrow \infty} e(t) = 0.\tag{28}$$

Theorem 1 *The adaptive estimator in (17) and (19)-(21) ensures*

$$\lim_{t \rightarrow \infty} \tilde{\theta}(t) = 0.\tag{29}$$

Proof of Theorem 1: Let $T' = \frac{2\pi}{\omega}$. It follows from (28) that

$$\lim_{t \rightarrow \infty} (\hat{\theta}(t + \tau) - \hat{\theta}(t)) = 0, \quad \forall \tau \in [0, T'].\tag{30}$$

From (28) and (30), we note that for arbitrary small $\epsilon_1 > 0$ and $\epsilon_2 > 0$, there exists $T > 0$ such that

$$\begin{aligned}\|e(t)\| &\leq \epsilon_1, \quad t \geq T, \\ \|\hat{\theta}(t + \tau) - \hat{\theta}(t)\| &\leq \epsilon_2, \quad t \geq T, \quad \tau \in [0, T'].\end{aligned}\tag{31}$$

We prove (29) by contradiction. If (29) is not true, it implies that for any $T > 0$, there exists $t' > T$ and finite $\epsilon_3 > 0$ such that

$$|\tilde{\theta}(t')| \geq \epsilon_3.\tag{32}$$

It follows from (31) and (32) that there exists finite $\epsilon_3 > 0$ and t' such that

$$\|e(t)\| \leq \epsilon_1, \quad t \geq t',\tag{33}$$

$$\|\hat{\theta}(t' + \tau) - \hat{\theta}(t')\| \leq \epsilon_2, \quad \tau \in [0, T']\tag{34}$$

$$|\tilde{\theta}(t')| \geq \epsilon_3,\tag{35}$$

for arbitrary small ϵ_1 and ϵ_2 . Since F_a is nonsingular and the choice of $u_x(t)$ ensures that the rows of $F_c(t)$ are linearly independent over any interval T' , we note that for any t' and $\tilde{\theta} \neq 0$, there exists $t_1 \in [t', t' + T']$ and $\eta > 0$ such that

$$\|F_a(t_1)F_c^\top(t_1)\tilde{\theta}\| \geq \eta\|\tilde{\theta}\|.\tag{36}$$

From (34), (35) and (37), we have

$$\|F_a(t_1)F_c^\top(t_1)\tilde{\theta}(t_1)\| \geq \|F_a(t_1)F_c^\top(t_1)\tilde{\theta}(t_1)\| \geq \eta(\|\tilde{\theta}(t')\| - \epsilon_2) \geq \eta(\epsilon_3 - \epsilon_2). \quad (37)$$

We note that the rate of change of $F_\theta(t) = \|F_a(t)F_c^\top(t)\tilde{\theta}(t)\|$ is bounded, and we denote $U = \sup_{t \geq 0} |\dot{F}_\theta(t)|$. Let

$$\bar{T} = \frac{\eta(\epsilon_3 - \epsilon_2) - \lambda_{\max}(A_m)\epsilon_1}{U},$$

where $\lambda_{\max}(A_m)$ is the maximum eigenvalue of A_m . It follows from (23) that over $\tau \in [0, \bar{T}]$,

$$\dot{e}(t_1 + \tau) = F_\theta(t_1) + \int_{t_1}^{t_1 + \tau} \dot{F}_\theta(\tau) d\tau + A_m e(t_1 + \tau),$$

and hence the relationship in (33) implies that

$$\left\| \int_{t_1}^{t_1 + \bar{T}} \dot{e}(\tau) d\tau \right\| \geq \|F_\theta(t_1)\| \bar{T} - \frac{U}{2} \bar{T}^2 - \lambda_{\max}(A_m)\epsilon_1 \bar{T}.$$

Since

$$\|e(t_1 + \bar{T})\| \geq \left\| \int_{t_1}^{t_1 + \bar{T}} \dot{e}(\tau) d\tau \right\| - \|e(t_1)\|,$$

it follows from (33) and (37) that

$$\|e(t_1 + \bar{T})\| \geq (\eta(\epsilon_3 - \epsilon_2) - \lambda_{\max}(A_m)\epsilon_1)\bar{T} - \frac{U}{2}\bar{T}^2 - \lambda_{\max}(A_m)\epsilon_1. \quad (38)$$

For any finite ϵ_3 , we can always find ϵ_1, ϵ_2 such that

$$\|e(t_1 + \bar{T})\| \geq \epsilon_1,$$

which contradicts (33). Hence, (29) holds, and this completes the proof. $\square$

Corollary 1

$$\begin{aligned} \lim_{t \rightarrow \infty} \hat{L}_e(t) &= L, \\ \lim_{t \rightarrow \infty} \hat{V}_{le}(t) &= V_L, \\ \lim_{t \rightarrow \infty} \hat{\phi}_L(t) &= \phi_L. \end{aligned}$$

Proof of Corollary 1: It follows from Theorem 1 and (24) that

$$\begin{aligned} \lim_{t \rightarrow 0} \hat{L}(t) &= L, \\ \lim_{t \rightarrow 0} \hat{V}_L(t) &= V_L. \end{aligned}$$

Therefore, Corollary 1 follows immediately from (25) and the fact that $C_1(0) = C_{2_1}(0) = C_{2_2}(0) = 1$. $\square$

Corollary 1 implies that $\hat{L}_e(t)$ and $\hat{V}_{le}(t)$ estimate the unknown parameters asymptotically. We further note that they are low-pass signals that can be used directly as inputs to the guidance law of the UAV.

IV. UAV's Guidance Law

While the active adaptive estimation system provides convergent estimates $\hat{L}_e(t)$ and $\hat{V}_{le}(t)$, the objective is to control the velocity $V_c(t)$ and the yaw rate $V_\psi(t)$ of the follower UAV to achieve the asymptotic convergence properties stated in (12)-(14), Figure 4. Since we argue that estimation can be decoupled from feedback control, the guidance law will be first designed for known L and ϕ_L . Then we prove that the convergence properties of the guidance law are not affected if the true values of L and ϕ_L are replaced by the filtered estimates of $\hat{L}_e(t)$ and $\hat{\phi}_L(t)$ from (25).

IV.A. Guidance Law Design

We note that

$$r_f(t) = \begin{bmatrix} r_{f_x}(t) \\ r_{f_y}(t) \end{bmatrix} = \begin{bmatrix} \frac{L}{2 \tan(\alpha(t)/2)} \sin(\beta(t)) \\ \frac{L}{2 \tan(\alpha(t)/2)} \cos(\beta(t)) \end{bmatrix}. \quad (39)$$

Assuming L and ϕ_L are available, the guidance law can be defined via the simplest proportional-integral structure:

$$\begin{aligned} V_\psi(t) &= -k_1(\psi(t) - \phi_L), \\ V_c(t) &= \begin{bmatrix} k_2(r_{f_x}(t) - r_{r_x}) \\ k_3(r_{f_y}(t) - r_{r_y}) + k_4 \int_0^t (r_{f_y}(\tau) - r_{r_y}) d\tau \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} r_f(t) u_\psi(t), \end{aligned} \quad (40)$$

where $r_f(t)$ is defined via (39), k_i , $i = 1, 2, 3, 4$ are design gains such that $k_1 > 0$, $k_2 > 0$, and $\begin{bmatrix} 0 & 1 \\ -k_4 & -k_3 \end{bmatrix}$ is Hurwitz.

Lemma 1 *Assuming L and ϕ_L are known, the guidance law in (40) ensures that the relationships in (12)-(14) hold.*

Proof of Lemma 1: Since

$$\dot{r}(t) = V_L(t) - V_F(t),$$

it follows from (1) and (4) that

$$\begin{aligned} \dot{r}_f(t) &= T_f(\psi(t)) \dot{r}(t) + \dot{\psi}(t) \frac{\partial T_f(\psi)}{\partial \psi} r(t) \\ &= T_f(\psi(t))(V_L(t) - V_F(t)) + u_\psi(t) \frac{\partial T_f(\psi)}{\partial \psi} T_f^{-1}(\psi(t)) r_f(t). \end{aligned}$$

It follows from (2) that

$$\begin{aligned} \frac{\partial T_f(\psi)}{\partial \psi} T_f^{-1}(\psi) &= \begin{bmatrix} \cos(\psi) & -\sin(\psi) \\ \sin(\psi) & \cos(\psi) \end{bmatrix} \begin{bmatrix} \sin(\psi) & -\cos(\psi) \\ \cos(\psi) & \sin(\psi) \end{bmatrix} \\ &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \end{aligned}$$

and Eq. (3) implies that

$$\dot{r}_f(t) = -V_c + T_f(\psi(t))V_L + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} r_f(t) u_\psi(t).$$

Incorporating the guidance law from (40), we have

$$\dot{r}_f(t) = - \begin{bmatrix} k_2(r_{f_x}(t) - r_{r_x}) \\ k_3(r_{f_y}(t) - r_{r_y}) + k_4 \int_0^t (r_{f_y}(\tau) - r_{r_y}) d\tau \end{bmatrix} + T_f(\psi(t))V_L. \quad (41)$$

Letting $\chi(t) = [\psi(t) \quad r_{f_x}(t) \quad \int_0^t (r_{f_y}(\tau) - r_{r_y}) d\tau \quad r_{f_y}(t)]^\top$, we have from (41) the following state-space representation:

$$\dot{\chi}(t) = A\chi(t) + b(t), \quad (42)$$

where

$$A = \begin{bmatrix} -k_1 & 0 & 0 & 0 \\ 0 & -k_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -k_4 & -k_3 \end{bmatrix}, \quad (43)$$

$$b(t) = \begin{bmatrix} k_1 \phi_L \\ k_2 r_{r_x} + \cos(\psi(t))V_{L_x} + \sin(\psi(t))V_{L_y} \\ r_{r_y} \\ k_3 r_{r_y} - \sin(\psi(t))V_{L_x} + \cos(\psi(t))V_{L_y} \end{bmatrix}. \quad (44)$$

Since r_r , V_L and ϕ_L are bounded, $b(t)$ is bounded. The choice of k_1, k_2, k_3, k_4 implies that A is Hurwitz, and hence the entire system is stable.

Now we consider the asymptotic performance of the entire system with the guidance law (40). We note that the first equation is decoupled from the rest and it simply implies that

$$\lim_{t \rightarrow \infty} \psi(t) = \phi_L.$$

It can be verified easily that

$$T(\phi_L)V_L = \begin{bmatrix} 0 \\ \|V_L\| \end{bmatrix} \quad (45)$$

and hence

$$\lim_{t \rightarrow \infty} b(t) = \begin{bmatrix} k_1 \phi_L \\ k_2 r_{r_x} \\ r_{r_y} \\ k_3 r_{r_y} + \|V_L\| \end{bmatrix}.$$

Applying the end value theorem, we have

$$\lim_{t \rightarrow \infty} \chi(t) = -A^{-1}(\lim_{t \rightarrow \infty} b(t)) = \begin{bmatrix} \phi_F \\ r_{r_x} \\ \|V_L\|/k_4 \\ r_{r_y} \end{bmatrix}. \quad (46)$$

It follows from (46) that

$$\lim_{t \rightarrow \infty} \psi(t) = \phi_L, \quad (47)$$

$$\lim_{t \rightarrow \infty} r_f(t) = r_r, \quad (48)$$

$$\lim_{t \rightarrow \infty} \int_0^t (r_{f_y}(\tau) - r_{r_y})d\tau = \|V_L\|/k_4 \quad (49)$$

From (40) and (47) it follows that

$$\lim_{t \rightarrow \infty} V_\psi(t) = 0$$

and hence (48)-(49) imply that

$$\lim_{t \rightarrow \infty} V_c(t) = \begin{bmatrix} 0 \\ \|V_L\| \end{bmatrix}. \quad (50)$$

It follows from (3), (45) and (50) that

$$\lim_{t \rightarrow \infty} V_F(t) = V_L, \quad (51)$$

and hence

$$\lim_{t \rightarrow \infty} \phi_F(t) = \phi_L, \quad (52)$$

which implies that (X_f, Y_f) and (X_l, Y_l) have the same coordinate transformation matrix and

$$\lim_{t \rightarrow \infty} (r_f(t) - r_l(t)) = 0. \quad (53)$$

It follows from (48) and (53) that

$$\lim_{t \rightarrow \infty} r_f(t) = r_r. \quad (54)$$

Eqs. (47), (52) and (54) verify the control objective in (12)-(14). $\square$

IV.B. Guidance Law using Estimation Results

Since L and ϕ_F are not available, we use $\hat{L}_e(t)$ and $\hat{\phi}_L(t)$ from the adaptive visual estimation algorithm. The resulting guidance law is the same, except for L and ϕ_F being replaced by $\hat{L}_e(t)$ and $\hat{\phi}_L(t)$ in (39) and (40). Thus, the complete guidance laws are:

$$\begin{aligned} V_\psi(t) &= -k_1(\psi(t) - \hat{\phi}_L(t)), \\ V_c(t) &= \begin{bmatrix} k_2(\hat{r}_{f_x}(t) - r_{r_x}) \\ k_3(\hat{r}_{f_y}(t) - r_{r_y}) + k_4 \int_0^t (\hat{r}_{f_y}(\tau) - r_{r_y}) d\tau \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \hat{r}_f(t) V_\psi(t). \end{aligned} \quad (55)$$

where

$$\hat{r}_f(t) = \begin{bmatrix} \hat{r}_{f_x}(t) \\ \hat{r}_{f_y}(t) \end{bmatrix} = \begin{bmatrix} \frac{\hat{L}_e(t)}{2 \tan(\alpha(t)/2)} \sin(\beta(t)) \\ \frac{\hat{L}_e(t)}{2 \tan(\alpha(t)/2)} \cos(\beta(t)) \end{bmatrix}. \quad (56)$$

In the following Theorem, we prove that replacing the true values by the estimated ones will not affect the stability and the asymptotic performance of the guidance law.

Theorem 2 *The adaptive estimator in (17) and (19)-(21) and the guidance law in (57) ensure that the asymptotic properties (12)-(14) hold.*

Proof of Theorem 2: Let

$$\begin{aligned} \tilde{\phi}_L &= \hat{\phi}_L(t) - \phi_L \\ \tilde{r}(t) &= [\tilde{r}_x(t) \quad \tilde{r}_y(t)]^\top = \hat{r}(t) - r_f(t). \end{aligned}$$

The controllers in (57) can be rewritten as:

$$\begin{aligned} V_\psi(t) &= k_1(\psi(t) - \phi_L - \tilde{\phi}_L(t)), \\ V_c(t) &= \begin{bmatrix} k_2(r_{f_x}(t) + \tilde{r}_{f_x}(t) - r_{r_x}) \\ k_3(r_{f_y}(t) + \tilde{r}_{f_y}(t) - r_{r_y}) + k_4 \int_0^t (r_{f_y}(\tau) + \tilde{r}_{f_y}(\tau) - r_{r_y}) d\tau \end{bmatrix} \\ &\quad - \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} (r_f(t) + \tilde{r}_f(t)) V_\psi(t). \end{aligned} \quad (57)$$

Letting $\chi(t) = [\psi(t) \quad r_{f_x}(t) \quad \int_0^t (r_{f_y}(\tau) - r_{r_y}) d\tau \quad r_{f_y}(t)]^\top$, similar to derivations in Lemma 1, leads to

$$\dot{\chi}(t) = A\chi(t) + \hat{b}(t) \quad (58)$$

where A is defined in (43) and

$$\hat{b}(t) = \begin{bmatrix} k_1\phi_L + \tilde{\psi}_l(t) \\ k_2r_{r_x} + \cos(\psi(t))V_{L_x} + \sin(\psi(t))V_{L_y} - \tilde{r}_f(t)V_\psi(t) \\ r_{r_y} \\ k_3r_{r_y} - \sin(\psi(t))V_{L_x} + \cos(\psi(t))V_{L_y} + \tilde{r}_f(t)V_\psi(t) \end{bmatrix}.$$

It can be verified easily from the adaptive estimator that $\tilde{\psi}_l(t)$ and $\tilde{r}_f(t)$ are bounded, and therefore Corollary 1 implies that

$$\begin{aligned} \lim_{t \rightarrow \infty} \tilde{\psi}_l(t) &= 0, \\ \lim_{t \rightarrow \infty} \tilde{r}_f(t) &= 0. \end{aligned} \quad (59)$$

Bounded $\tilde{\psi}_l(t)$ implies that $V_\psi(t)$ is bounded, and hence $\tilde{r}_f(t)V_\psi(t)$ is bounded. Boundedness of $\hat{b}(t)$ can be proved similar to boundedness of $b(t)$ in Lemma 1, and therefore substitution of L and ϕ_L by their estimates will not affect the stability of the entire system. It follows from (59) that

$$\lim_{t \rightarrow \infty} (\hat{b}(t) - b(t)) = 0.$$

Using the same methodology as in Lemma 1, (12)-(14) can be proved. $\square$

IV.C. Sensitivity Analysis

In this section, we discuss the sensitivity of visual measurements w.r.t. unknown leader's length L . For simplicity, we assume that in steady state $V_F = V_L$ and $\phi_F = \phi_L = 0$. It follows from (9) that

$$\dot{\gamma} = F_a(\gamma) \begin{bmatrix} 0 \\ -v \sin(\omega t) \frac{1}{L} \end{bmatrix},$$

which can be simplified as:

$$\dot{\gamma} = - \begin{bmatrix} 4 \sin^2(\alpha/2) \sin(-\beta) \\ 2 \tan(\alpha/2) \cos(-\beta) \end{bmatrix} v \sin(\omega t) \frac{1}{L}. \quad (60)$$

Let $\gamma_n = [\alpha_n \ \beta_n]^\top$ be the vector of visual measurements without any camera movement, i.e. when $u_x = 0$. It follows from (60) that

$$\gamma \approx \gamma_n + \begin{bmatrix} 4 \sin^2(\alpha_n/2) \sin(-\beta_n) \\ 2 \tan(\alpha_n/2) \cos(-\beta_n) \end{bmatrix} \frac{v}{\omega} \frac{1}{L} \cos(\omega t),$$

which along with (18) imply that

$$\gamma \approx \gamma_n + \begin{bmatrix} 4 \sin^2(\alpha_n/2) \sin(-\beta_n) \\ 2 \tan(\alpha_n/2) \cos(-\beta_n) \end{bmatrix} \frac{l}{L} \cos(\omega t). \quad (61)$$

We note that

$$\sin(\alpha_n/2) = \frac{L}{2R}, \quad \tan(\alpha_n/2) = \frac{L}{\sqrt{R^2 - L^2}}.$$

Therefore, Eq. (61) implies that

$$\gamma(t) \approx \gamma_n + \begin{bmatrix} \frac{lL}{R_n^2} \sin(\beta_n) \\ \frac{l}{\sqrt{R_n^2 - L^2}} \cos(\beta_n) \end{bmatrix} \cos(\omega t).$$

Thus, the active movement of camera leads to the following increment in the visual measurements:

$$\gamma_v(t) = \gamma(t) - \gamma_n(t) \approx \begin{bmatrix} \frac{lL}{R_n^2} \sin(\beta_n) \\ \frac{l}{\sqrt{R_n^2 - L^2}} \cos(\beta_n) \end{bmatrix} \cos(\omega t).$$

Let

$$\gamma_a = \begin{bmatrix} \frac{lL}{R_n^2} \sin(\beta_n) \\ \frac{l}{\sqrt{R_n^2 - L^2}} \cos(\beta_n) \end{bmatrix},$$

and hence the sensitivity of visual measurement w.r.t. L can be characterized as follows:

$$\frac{\partial \gamma_a}{\partial L} = \begin{bmatrix} \frac{l}{R_n^2} \sin(\beta_n) \\ \frac{2lL}{R_n^2 - L^2} \cos(\beta_n) \end{bmatrix}.$$

Given a pre-specified level of precision in visual measurements, larger $\frac{\partial \gamma_a}{\partial L}$ implies higher precision in estimation of L . We note that $\frac{\partial \gamma_a}{\partial L}$ is larger if l is larger, L is larger or R_n is smaller. We also note that $\frac{\partial \gamma_a}{\partial L}$ depends also on bearing angle β_n .

V. Simulations

Let the lead aircraft have unknown characteristic length $L = 5$ and velocity $V_L = [75 \ 75]^\top$. Let the desired separation be $r_r = [-10 \ -10]^\top$. The original states of the follower are initialized as:

$$\begin{aligned} \phi(0) &= 0.5 \\ V_2(0) &= 50 \\ V_1(0) &= 0. \end{aligned}$$

Figure 6. $\hat{L}_e(t)$ (solid) and $\hat{V}_{le}(t)$ (dashed)

In the active adaptive visual estimation system, we set

$$A_m = \begin{bmatrix} 0 & 1 \\ -1 & -1.4 \end{bmatrix},$$

$$Q = \begin{bmatrix} 200 & 0 \\ 0 & 200 \end{bmatrix}.$$

and let the adaptive gain be

$$\Gamma = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}.$$

The open-control signal for the active camera is set to:

$$u_x = 0.5 \sin(20t).$$

For the guidance law design, we set

$$\begin{aligned} k_1 &= 1, \\ k_2 &= 1, \\ k_3 &= 1.4, \\ k_4 &= 1, \end{aligned}$$

Figure 6 shows the low-pass filtered estimation of $\hat{L}_e(t)$ and $\hat{V}_{le}(t)$. In Figure 7, the yaw rate $\psi(t)$ is plotted, which is shown to converge to the leader's heading angle $\pi/4$. In Figure 8, the longitudinal $V_2(t)$ and the lateral $V_1(t)$ velocities are plotted. We note that $V_1(t)$ converges to zero upon the transient. The follower's velocity $V_F(x)$ is plotted in Figure 9, which is shown to converge to the leader's velocity V_L . The relative separation $r(t)$ of the two aircraft in (X_f, Y_f) space is plotted in Figure 10, which is shown to arrive to the desired separation r_r . The phase plot of $r(t)$ is illustrated in Figure 11.

We further assume the leader changes its velocity to $V_L = [85 \ 65]^\top$ at the time instant $t = 20\text{sec.}$ Figure 12 plots $\hat{L}_e(t)$ and $\hat{V}_{le}(t)$, while Figure 13 shows the yaw rate $\psi(t)$. In Figure 14, longitudinal and lateral velocities $V_2(t)$ and $V_1(t)$ are plotted, and the follower's velocity $V_F(x)$ is plotted in Figure 15. The relative separation $r(t)$ of the two aircraft in (X_f, Y_f) space is plotted in Figure 16, and the phase plot of $r(t)$ is illustrated in Figure 11. We notice that the guidance law enables the follower UAV track the maneuvering leader with the help of the active adaptive visual estimation system.

Figure 7. $\psi(t)$ (solid) and $\phi_L = \pi/4$ (dashed)

Figure 8. $V_{cx}(t)$ (solid) and $V_{cy}(t)$ (dashed)

VI. Conclusion

In this paper, an active adaptive vision estimation system is proposed. By active open-loop control of the camera, low-pass filtered estimates of the unknown parameters are obtained which converge to the true parameters asymptotically. The guidance law uses these estimates directly to maintain the desired lateral separation between the leader and the follower. Simulation results for both constant and time-varying velocities of the leader demonstrate the effectiveness of the estimation and control algorithms. Similar to Refs, ^{16,17} one can consider a fast adaptation scheme leading to desired estimation precision in the transient phase. This idea will be explored in our future papers.

Figure 9. $V_{Fx}(t)$ (solid) and $V_{Fy}(t)$ (dashed)

Figure 10. $r(t) = [r_x(t) \ r_y(t)]^T$ (solid) and r_r (dashed)

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Figure 11. $r(t) = [r_x(t) \ r_y(t)]^\top$ in $\mathbb{R}^2$ space as trajectory of time

Figure 12. $\hat{L}_e(t)$ (solid) and $\hat{V}_{le}(t)$ (dashed)

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Figure 14. $V_{cx}(t)$ (solid) and $V_{cy}(t)$ (dashed)

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Figure 15. $V_{F_x}(t)$ (solid) and $V_{F_y}(t)$ (dashed)

Figure 16. $r(t) = [r_x(t) \ r_y(t)]^T$ (solid) and r_r (dashed)

Figure 17. $r(t) = [r_x(t) \ r_y(t)]^\top$ in $\mathbb{R}^2$ space as trajectory of time