An $\mathcal{L}_1$ Adaptive Pitch Controller for Miniature Air Vehicles

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One of the challenges in designing low level control loops for Micro Air Vehicles (MAVs) is that the manufacturing process for airframes is not consistent enough to ensure uniform aerodynamic properties. Therefore, there is a significant need for robust adaptive control techniques that are computationally simple. Conventional Model Reference Adaptive Controllers (MRAC) have proved to be very useful in a number of flight tests over the past years. However, a major drawback of this control architecture is that during the transient the control signal or the system output can exhibit large oscillations. This requires intensive Monte-Carlo testing for all possible variations in all possible scenarios before each flight test. This paper presents preliminary results for a novel adaptive control technique that is both computationally simple, and has uniform bounded transient response. The effectiveness of the proposed control scheme is demonstrated through simulation results produced by a medium-fidelity hardware-in-the-loop simulator as well as flight test results on a five foot wingspan unmanned air vehicle.

I. Introduction

The availability of small low power microprocessors and advances in solid state sensor technology have enabled the development of small autopilots that have made it feasible to develop autonomous micro air vehicles (MAVs).^{1,2} However, for MAVs to be economically viable, the cost of the electronics and the airframe must be significantly less than conventional UAVs. There are currently a large variety of MAV airframes, many of which are modified RC hobby aircraft. For most of these airframes, the aerodynamic coefficients have not been suitably characterized and doing so is not economically feasible. The manufacturing processes associated with MAV airframes are not precise enough to guarantee uniformity in the aerodynamic coefficients. In addition, MAVs are designed to withstand frequent crashes, however the crashes often change the aerodynamic properties of the airframe.

Therefore, the design of control systems for MAV cannot depend on accurate knowledge of the aerodynamic coefficients. An obvious solution is to use adaptive control techniques for the inner loops of the autopilot. A variety of adaptive control techniques have been proposed for air vehicles including, neural networks, least squares estimation, and Lyapunov based methods. The neural network approach typically entails training a network off-line for model inversion, and then using an on-line adaptive network to compensate for modeling errors.³⁻⁷ Recursive least squares techniques identify the airframe parameters on-line and use these parameters to adjust the controller. Such controllers have the ability to quickly converge on the airframe parameters,⁸ and therefore adapt quickly to failures. Other approaches to aircraft autopilot design involving least squares estimation.^{6,9,10} In Lyapunov based approaches, the parameter update law is selected to ensure stability of the tracking error, but generally does not ensure parameter convergence.¹¹⁻¹⁴

Adaptive control for MAVs is still in its infancy. The Model Reference Adaptive Control (MRAC) methodology was recently used to design roll and pitch attitude hold loops for MAVs in Ref. 15,16. Initial flight tests demonstrated the feasibility of the method, however experimental implementation also highlighted

several difficulties.^{15,16} In particular, the MRAC controllers are relatively sensitive to the adaptive gain and proved difficult to tune experimentally. When the gain was improperly tuned, there were high frequency oscillations in the control signal which led to erratic flight behavior.

In Refs. 17, 18, a novel $\mathcal{L}_1$ adaptive control architecture has been developed which guarantees smooth tracking for both system input and output signals. A significant feature of the $\mathcal{L}_1$ adaptive framework is that oscillations in the control effort are effectively eliminated, and the method is easy to tune. The $\mathcal{L}_1$ adaptive control architecture replaces the conventional MRAC by an equivalent companion model architecture first, which further enables insertion of a low-pass filter in the closed-loop. To ensure asymptotic stability of the closed-loop system the $\mathcal{L}_1$ -gain of the cascaded system, comprised of the low-pass filter and the desired closed-loop reference system needs to be less than the inverse of the upper bound on the unknown parameters used in projection-type adaptation law.¹⁷ Design guidelines for selection of the low-pass filter that ensure desired transient response are derived in Ref 18.

The objective of this paper is to explore the application of the $\mathcal{L}_1$ adaptive control methodology to the pitch attitude hold loop for MAVs. Section II derives a simple model for the pitch attitude hold loop. Section III derives an MRAC controller and demonstrates some of the difficulties encountered with that method. Section V describes the application of the $\mathcal{L}_1$ adaptive controller to the pitch attitude hold loop. Simulation results are given in Section VI and flight tests results on a five foot wingspan unmanned air vehicle are given in Section VII describes .

II. Mathematical Model for MAV Pitch Dynamics

Under standard symmetry assumptions on the airframe, the inertia matrix has the form

$$\mathbf{J} = \begin{pmatrix} J_x & 0 & -J_{xz} \\ 0 & J_y & 0 \\ -J_{xz} & 0 & J_z \end{pmatrix}.$$

For MAVs, the aerodynamic moments are adequately modeled as linear in the pitch rate q and the elevator command δ_e . Therefore, the pitch dynamics of the MAV are given by

$$\dot{\theta} = q \cos \phi - r \sin \phi \quad (1)$$

$$\dot{q} = \frac{J_{xz}}{J_y}(r^2 - p^2) + \frac{J_z - J_x}{J_y}pr + \frac{1}{2J_y}\rho V^2 \bar{c}S \left[C_m(\alpha) + C_{m_q} \frac{\bar{c}q}{V} + C_{m_{\delta_e}} \delta_e \right] \quad (2)$$

where θ is the pitch angle, ϕ is the roll angle, p , q , and r , are the roll, pitch, and yaw rates respectively, V is the airspeed, S is the wing area, b is the wing span, $\bar{c}$ is the average cord length, C_* are the aerodynamic coefficients, and α is the angle of attack.

Throughout the paper, we make the following assumptions:

A1: V , θ , and ϕ can be measured or adequately estimated by on-board sensors,

A2: The pitch angle is limited to $-\bar{\theta} \leq \theta \leq \bar{\theta}$ where $\bar{\theta} < \pi/2$,

A3: The roll angle is limited to $-\bar{\phi} \leq \phi \leq \bar{\phi}$ where $\bar{\phi} < \pi/2$.

Note that we do not assume that the angle of attack α and the pitch rate q are measured. Following Ref. 15 we reduce (1)-(2) by solving (2) for q to obtain

$$q = \frac{\dot{q} + \frac{J_{xz}}{J_y}(p^2 - r^2) + \left(\frac{J_x - J_z}{J_y} \right) pr}{\left(\frac{\rho V \bar{c} S C_{m_q}}{2J_y} \right)} - \frac{VC_m(\alpha)}{\bar{c}C_{m_q}} - \frac{VC_{m_{\delta_e}}}{\bar{c}C_{m_q}} \delta_e,$$

and substituting into (1) to get

$$\begin{aligned} \dot{\theta} &= \left[\frac{\dot{q} + \frac{J_{xz}}{J_y}(p^2 - r^2) + \left(\frac{J_x - J_z}{J_y} \right) pr}{\left(\frac{\rho V \bar{c} S C_{m_q}}{2J_y} \right)} - \frac{VC_m(\alpha)}{\bar{c}C_{m_q}} - \frac{VC_{m_{\delta_e}}}{\bar{c}C_{m_q}} \delta_e \right] \cos \phi - r \sin \phi \\ &= -V \cos \phi K_1 + V \cos \phi K_2 \delta_e, \end{aligned}$$

where

$$K_1 \triangleq \left[\frac{\dot{q} + \frac{J_{xz}}{J_y}(p^2 - r^2) + \left(\frac{J_x - J_z}{J_y} \right) pr}{\left(\frac{\rho V^2 \bar{c} S C_{m_q}}{2 J_y} \right)} - \frac{C_m(\alpha)}{\bar{c} C_{m_q}} \right] - \frac{r}{V} \tan \phi.$$

$$K_2 \triangleq -\frac{C_{m\delta_e}}{\bar{c} C_{m_q}}.$$

Note that in wings level, constant altitude flight K_1 is a constant. We will make the assumption that K_1 is an unknown constant and use the adaptive algorithm to estimate K_1 . We also assume that K_2 is known. Our simplified model is therefore given by

$$\dot{\theta} = -V \cos \phi K_1 + V \cos \phi K_2 \delta_e. \quad (3)$$

The control objective is to force θ to track a bounded continuous reference signal θ^c both in transient and steady state.

III. Model Reference Adaptive Controller

Before deriving the $\mathcal{L}_1$ adaptive controller, we will derive the MRAC controller for this system. Consider the reference model

$$\dot{\theta}^m = -a\theta^m + a\theta^c, \quad (4)$$

where $a > 0$, and let $\tilde{\theta} \triangleq \theta - \theta^m$. Differentiating we get

$$\dot{\tilde{\theta}} = -V \cos \phi K_1 + V \cos \phi K_2 \delta_e + a(\theta^m - \theta^c).$$

If we knew K_1 then we would pick

$$\delta_e^d = \frac{K_1}{K_2} + \frac{a(\theta^c - \theta^m) + \lambda(\theta^m - \theta)}{V \cos \phi K_2}$$

Letting the control input be

$$\delta_e = \frac{\hat{K}_1}{K_2} + \frac{a(\theta^c - \theta^m) + \lambda(\theta^m - \theta)}{V \cos \phi K_2} \quad (5)$$

where $\hat{K}_1$ is the estimate of K_1 , we have

$$\dot{\tilde{\theta}} = -\lambda \tilde{\theta} + V \cos \phi (\hat{K}_1 - K_1).$$

Letting

$$V_m = \frac{1}{2} \tilde{\theta}^2 + \frac{1}{2\gamma} (\hat{K}_1 - K_1)^2,$$

and differentiating, we get

$$\dot{V}_m = -\lambda \tilde{\theta}^2 + (\hat{K}_1 - K_1) \left[(\theta - \theta^m) V \cos \phi + \frac{\dot{\hat{K}}_1}{\gamma} \right].$$

Therefore, selecting the adaptive gain as

$$\dot{\hat{K}}_1 = -\gamma \text{Proj} \left((\theta - \theta^m) V \cos \phi, \hat{K}_1 \right), \quad (6)$$

we can use standard arguments to show that $\theta \rightarrow \theta^m$.

IV. MRAC Simulation

To test the performance of the MRAC controller we have implemented a simulation using Simulink. We have implemented only the pitch dynamics, and used artificial variables for p , r , ϕ , and V . In particular we let

$$V = 10 + 4 \sin(2\pi t), \quad p = 0.2 \sin(2\pi 0.1 t), \quad r = 0.1 \sin(2\pi 0.1 t), \quad \phi = \int p.$$

The control gains used in the simulation were

$$a = .1, \quad \lambda = 1000, \quad \gamma = 10, \quad \bar{K}_1 = 1, \quad \underline{K}_1 = -1.$$

The Simulink diagram is shown in Figure 1.

Figure 1. Simulink diagram

The results over 200 seconds are shown in Figure 2. While these results look adequate, if we zoom in on the first second of the simulation we see unacceptable transient performance, as shown in Figure 3.

V. $\mathcal{L}_1$ Adaptive Controller

In this section we will derive the $\mathcal{L}_1$ adaptive controller. The simplified model of the pitching motion given in Equation (3) can be rewritten as

$$\dot{\theta} = -a\theta + a\theta - V \cos \phi K_1 + V \cos \phi K_2 \delta_e.$$

Instead of using the reference model in (4) we will use the companion model

$$\dot{\theta}^m = -a\theta^m + a\theta - V \cos \phi \hat{K}_1 + V \cos \phi K_2 \delta_e, \quad \theta^m(0) = \theta(0), \quad (7)$$

where $\hat{K}_1$ is the parameter estimate given by (6). Again, letting $\tilde{\theta} = \theta - \theta^m$ we get

$$\dot{\tilde{\theta}} = -a\tilde{\theta} + V \cos \phi (\hat{K}_1 - K_1).$$

Instead of using the controller given in (5) we let $\lambda = a$ and low pass filter $V \cos \phi \hat{K}_1$ to get

$$\delta_e = \frac{C(s)\{V \cos \phi \hat{K}_1\}}{V \cos \phi K_2} + \frac{a(\theta^c - \theta)}{V \cos \phi K_2}, \quad (8)$$

where we have used the notation $C(s)\{\xi\}$ to denote the low pass filtered version of ξ by $C(s)$. Following Ref. 18, we define the following reference system:

$$\dot{\theta}_r = a(\theta^c - \theta_r) - (1 - C(s))\{V \cos \phi K_1\} \quad \theta_r(0) = \theta(0). \quad (9)$$

The theory developed in Refs 17,18, when applied to this problem, leads to the following result.

Figure 2. Results for the MRAC control.

Theorem V.1 *The $\mathcal{L}_1$ adaptive controller in (6), (7) and (8) ensures*

$$\lim_{\gamma \rightarrow \infty} \|\theta - \theta_r\|_{\mathcal{L}_\infty} = 0. \quad (10)$$

Proof. Substituting (8) into (7), the closed-loop companion model is given by

$$\dot{\theta}^m = a(\theta^c - \theta^m) - (1 - C(s)) \{V \cos \phi \hat{K}_1\}. \quad (11)$$

Letting $\eta = \theta_m - \theta_r$ gives

$$\eta = \left(\frac{1}{s + a} (1 - C(s)) \right) \{V \cos \phi (\hat{K}_1 - K_1)\},$$

from Refs. 17, 18 we have

$$\lim_{\gamma \rightarrow \infty} \|\theta - \theta_m\|_{\mathcal{L}_\infty} = 0, \quad (12)$$

and hence

$$\lim_{\gamma \rightarrow \infty} \|\eta\|_{\mathcal{L}_\infty} = 0. \quad (13)$$

It follows from (9), (11) and (13) that

$$\lim_{\gamma \rightarrow \infty} \|\theta_m - \theta_r\|_{\mathcal{L}_\infty} = 0, \quad (14)$$

which along with (12) leads to (10). $\square$

Thus, using large adaptive gain, the resulting closed-loop adaptive control system follows a given reference system that depends upon the unknown parameter $K_1(t)$. If we choose the bandwidth of $C(s)$ larger than that of the signal $K_1(t)$, we can achieve $C(s) \{K_1\} \approx K_1$, thus minimizing the deviation of θ from desired θ_r due to unknown $K_1(t)$. We note that increasing the bandwidth of $C(s)$ leads to increased bandwidth of the control signal δ_e . Unlike MRAC, $\mathcal{L}_1$ adaptive controller can control this trade-off in the design phase as well as guarantee the transient performance.

Figure 3. Results for the MRAC controller during the first second of the simulation.

VI. $\mathcal{L}_1$ Control Simulation

This section gives simulation results for the $\mathcal{L}_1$ adaptive control derived in the previous section. A similar framework to that described in Section IV is used for the $\mathcal{L}_1$ adaptive controller. The low pass filter is given by

$$C(s) = \frac{\omega_0}{s + \omega_0}.$$

Simulation parameters are given by

$$a = .1, \quad \gamma = 10000, \quad \omega_0 = 160, \quad \overline{K}_1 = 1, \quad \underline{K}_1 = -1.$$

Simulation results are shown in Figure 4(a). Note that the transient behavior of θ does not exhibit oscillations. The oscillations in δ_e are actually due to the unnatural forced oscillations in V , p and ϕ . Figure 4(b) shows results if V , p , and ϕ are constant. In contrast, the MRAC controller exhibits extreme transients even when V , p and ϕ are constant.

(a) V, p, ϕ time-varying.

(b) V, p, ϕ constant.

Figure 4. Results for the $\mathcal{L}_1$ adaptive control.

VII. Experimental Flight Tests

A. Platform

The algorithms were flown on a micro air vehicle equipped with the Procerus Kestrel Autopilot² running a 29 MHz Rabbit microcontroller with 512K Flash and 512K RAM. The sensors on the autopilot include rate gyros, accelerometers, an absolute pressure sensor for measuring altitude, a differential pressure sensor for measuring airspeed, and a GPS receiver. A two state extended Kalman filter (EKF) is used to estimate the roll and pitch angles. The state equations of the EKF are propagated using the rate gyros and the accelerometers are used for the measurement update. The autopilot was programmed in C. The adaptive control algorithm was programmed in Dynamic C (a variant of C) and runs at a variable sample rate of approximately 80-90 hertz. The Kestrel autopilot is shown in Figure 5(a). The airframe, shown in Figure 5(b) was a custom BYU designed flying wing with a 54 inch wingspan, a cruise speed of 17 m/s, and a take-off weight of 3 lbs. The airframe is powered by a 12 Volt brushless electric motor, powered by four lithium-polymer batteries. The airframe is hand launched and belly landed.

(a) The Procerus Kestrel autopilot used to demonstrate the $\mathcal{L}_1$ adaptive control algorithm.

(b) The airframe used to demonstrate the $\mathcal{L}_1$ adaptive control algorithm.

B. Implementation Notes

To enhance the performance of the adaptive control algorithms we have added dead zone.¹⁹ With dead zone, adaptation is stopped when the actual value is within a certain region around the desired value. When in a flight pattern with little excitation, the parameters will adapt until the MAV is close to the desired value. Then adaptation freezes until flight conditions change such that we are no longer within the no-adaptation dead zone.

The $\mathcal{L}_1$ controller was first tuned on our 6 DOF simulator before flight testing. The simulator has flight characteristics that are very similar to the MAV shown in Figure 5(b). Theoretically, the $\mathcal{L}_1$ control can have a high adaptive gains without causing oscillations. However, in practice, the sample rate of the microcontroller limits the gain. Therefore, the $\mathcal{L}_1$ controller was tuned separately for the flight hardware.

C. Results

After tuning the adaptive control, the hourglass-shaped flight plan shown in Figure 5 was flown multiple times. The hourglass pattern was chosen because it requires both right and left turn of the UAV. An outerloop designed to follow waypoint paths is used to command the desired pitch.

Figure 5. Hourglass shaped flight pattern flown during flight tests, with commanded waypoints shown. The waypoints have different commanded altitudes.

Figure 6(a) shows the commanded pitch angle, the pitch angle produced by the companion model, and the actual pitch angle. The associated elevator command is shown in Figure 6(b), and the adaptive gain is shown in Figure 6(c).

Figure 7 shows the roll angle during the hourglass maneuver. For comparison, Figure 8 shows pitch tracking results for the hourglass maneuver using a well tuned PID controller. Notice that the PID controller tracks positive pitch values well, but it does not track negative pitch values very well. This is due to the aircraft picking up speed with negative pitch, thus generating more lift and introducing steady state error.

(a) Pitch

(b) Elevator

(c) Adaptive gain

Figure 6. Flight results using the $\mathcal{L}_1$ adaptive control scheme. (a) The commanded pitch, companion model pitch, and actual pitch angle produced while flying the hourglass pattern. Notice around 50 sec. the companion model diverges for about 10 seconds due to a communication event that significantly reduced the sample rate. A 30 degree roll causes the anomalies at 67 sec. and 92 sec. (b) The elevator command produced by the $\mathcal{L}_1$ adaptive control while flying the hourglass pattern. (c) The adaptive gain produced by the $\mathcal{L}_1$ adaptive control while flying the hourglass pattern.

Figure 7. PID roll when $\mathcal{L}_1$ controller is in use. Notice how steep rolls of about 30 degrees effects pitch in Figure 6(a).

Figure 8. PID pitch control on Ray flying an hourglass pattern.

VIII. Conclusions

In this paper we have applied the recently introduced $\mathcal{L}_1$ adaptive control algorithm to pitch attitude control for small UAVs. The algorithm has been successfully demonstrated in both simulation and on actual flight hardware using the Kestrel autopilot. The $\mathcal{L}_1$ adaptive algorithm results in performance that exceeds the baseline PID controllers typically used on the autopilot and exhibits robustness to a variable sample rate for the processor, as well as the time delays introduced by state estimation. The algorithm also appears to be robust with respect to state estimation noise. Further research is needed to fully quantify the characteristics and benefits of the proposed algorithm.

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