Course Project: Inverted Pendulum Analysis

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1. Goal

Model an inverted pendulum and evaluate its dynamical characteristics.

2. Background

An inverted pendulum is a pendulum that has its major mass above its pivot joint. While a conventional pendulum is stable at its resting position (hanging downwards), an inverted pendulum is inherently unstable, thus requiring an active balancing mechanism to keep it stay upright. Such a feature makes it widely used as a benchmark for evaluating the performance of control algorithms.

Many real-world dynamic systems can be modeled as an inverted pendulum. An example is the Segway PT, as illustrated in Figure 1.

![Figure 1. Pictures of the Segway PT (Courtesy of Wikipedia)](image)

The objective of this course project is to model an inverted pendulum and analyze its dynamic characteristics, using the knowledge learned in the Linear System Theory class (ME3253).

3. Problem Statement

With simplification, an inverted pendulum resembling the Segway PT can be modeled as shown in Figure 2.
Figure 2. A schematic view of a simple inverted pendulum

The cart is assumed to be placed on a smooth, level surface where friction between the cart and the ground is negligible. The masses of the pendulum and the cart are represented by \( m \) and \( M \), respectively. The connecting rod is considered massless. The pivoting joint has a viscous friction coefficient of \( B \). Ideally, the pendulum would remain at the upright position, enabled by the external force input \( f(t) \), which actively balance the system.

4. Initial Condition

At time \( t = 0 \), \( \theta = 0.5^\circ \), and the velocity of the cart \( v = 0 \).

\[
M = 10 \text{ kg}, \ m = 1 \text{ kg}, \ g = 9.8 \text{ m/s}^2, \ L = 0.4 \text{ m}, \ W = 0.3 \text{ m}, \ H = 0.1 \text{ m}, \ B = 0.02 \text{ N·m·sec/rad}.
\]

5. Boundary Condition

\(-5^\circ < \theta < 5^\circ\)

(Approximation: when the angle \( \theta \) is within the range of \(-5^\circ < \theta < 5^\circ\), \( \sin \theta \approx \theta, \cos \theta \approx 1 \))

6. Tasks

1) Answer the question: If \( f(t) = 0 \), can the system be stable? Explain why.

2) Model the system using:
   - Free-body diagrams
   - State Variable and Input/Output equations. Take the external force \( f(t) \) as the input and the angle \( \theta \) as output.
   - Perform Laplace Transform and provide transfer function of the system.
   - Evaluate the stability of the system using built-in functions in MATLAB, e.g. Bode Plot, Nyquist Plot, etc.
3) With the transfer function of the feedback loop provided (by the course instructor), analyze the system’s dynamical characteristics:

- Draw block diagram and the overall transfer function for the closed-loop system.
- Evaluate the stability and relevant characteristics of the closed-loop system using MATLAB.
- Determine system response to a unit step function input using MATLAB. Take \( f(t) \) as the input and \( \theta \) as the output.
- Determine system response to an impulse function input using MATLAB. Take \( f(t) \) as the input and \( \theta \) as the output.

7. Suggested Work Flow

![Flowchart Diagram]

- Analyze the System
- Draw FBD
- State – Variable and Input/Output Representation
- Laplace Transform
- Transfer Function
- Evaluate System Characteristics
- Transfer Function of Feedback Block (to be provided)
- Block Diagram and Overall Transfer Function
- Evaluate Stability and other Characteristics of the System
- Step Response
- Impulse Response